



Mrs Angie Motshekga,
Minister of Basic
Education



Mr Enver Surty,
Deputy Minister
of Basic Education

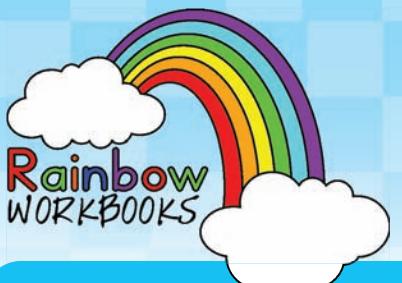
These workbooks have been developed for the children of South Africa under the leadership of the Minister of Basic Education, Mrs Angie Motshekga, and the Deputy Minister of Basic Education, Mr Enver Surty.

The Rainbow Workbooks form part of the Department of Basic Education's range of interventions aimed at improving the performance of South African learners in the first six grades. As one of the priorities of the Government's Plan of Action, this project has been made possible by the generous funding of the National Treasury. This has enabled the Department to make these workbooks, in all the official languages, available at no cost.

We hope that teachers will find these workbooks useful in their everyday teaching and in ensuring that their learners cover the curriculum. We have taken care to guide the teacher through each of the activities by the inclusion of icons that indicate what it is that the learner should do.

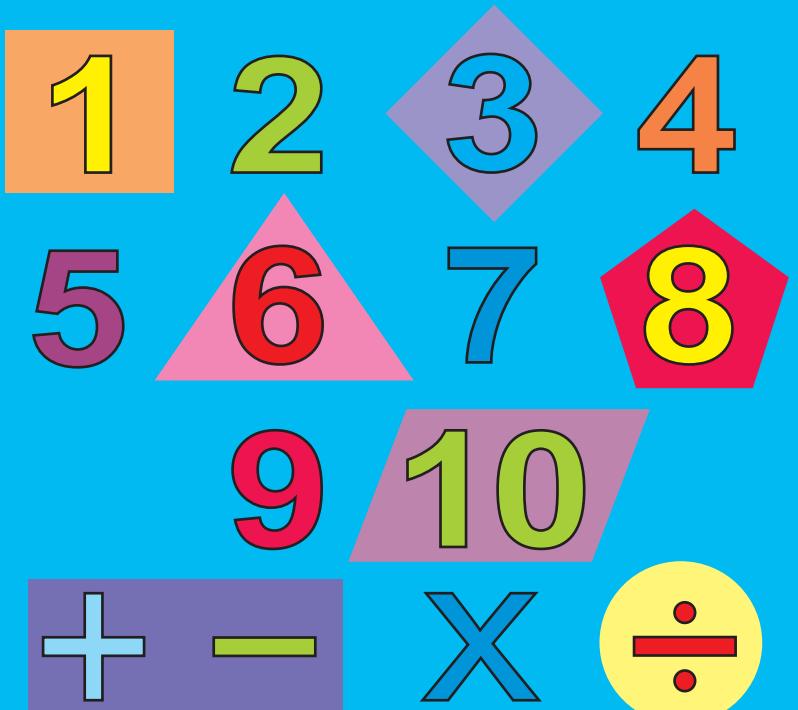
We sincerely hope that children will enjoy working through the book as they grow and learn, and that you, the teacher, will share their pleasure.

We wish you and your learners every success in using these workbooks.



MATHEMATICS IN ENGLISH
GRADE 7 - TERMS 3&4
ISBN 978-1-4315-0220-2

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MATHEMATICS in ENGLISH - Grade 7 Book 2



basic education
Department:
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REPUBLIC OF SOUTH AFRICA

Name:

Class:



MATHEMATICS in ENGLISH

Grade 7

Term 3&4



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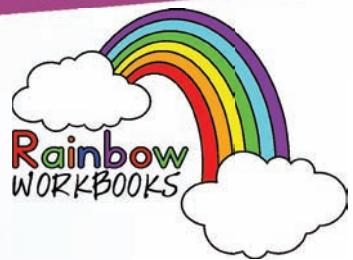
$$3 \times 4 = 12$$

Multiplication table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

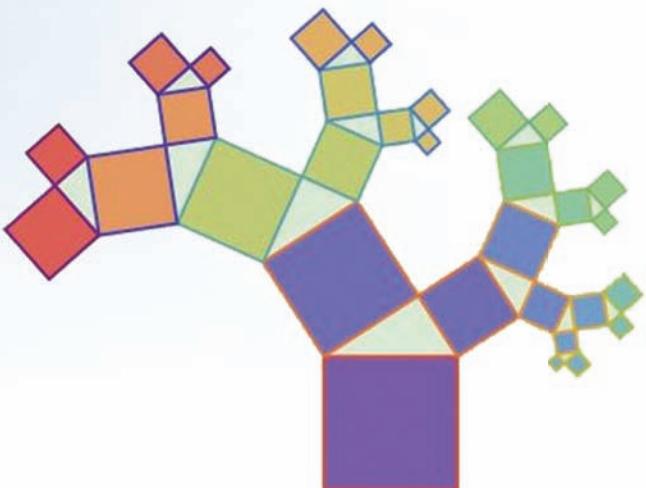


Grade 7



Mathematics

in ENGLISH



ENGLISH
Book 2

Name:



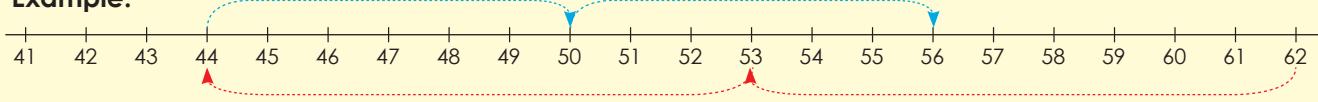
Numeric patterns: constant difference

We described the patterns using "adding" and "subtracting". Discuss..

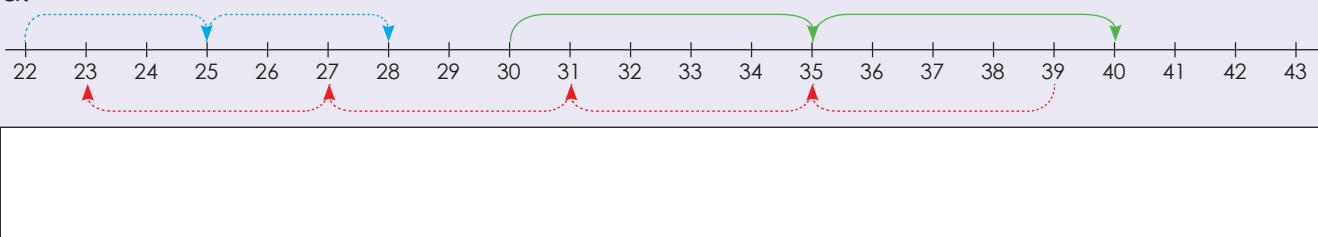


1. Describe each pattern.

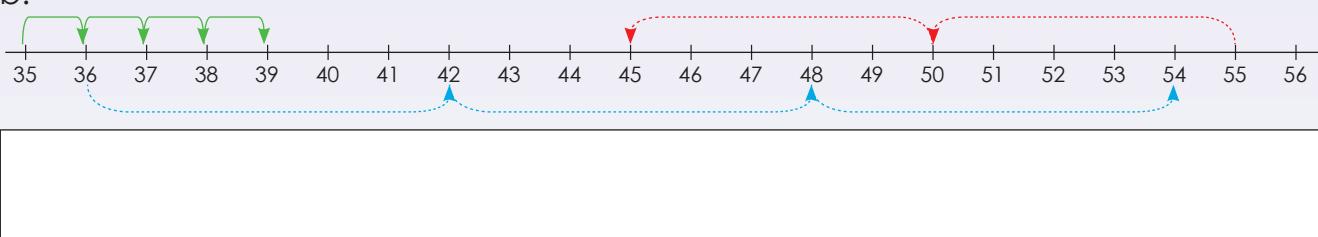
Example:



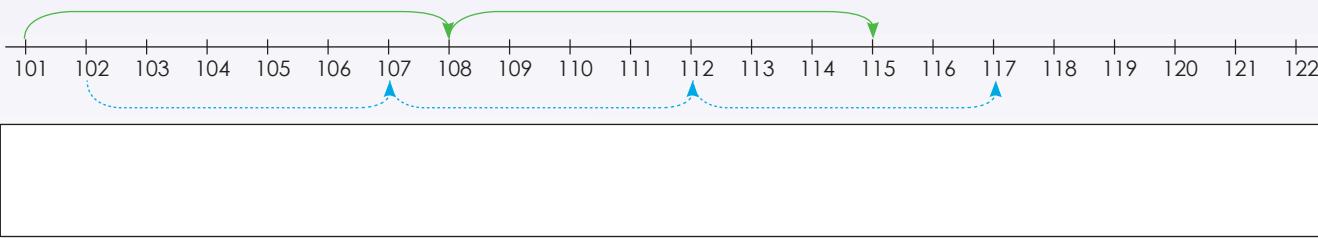
a.



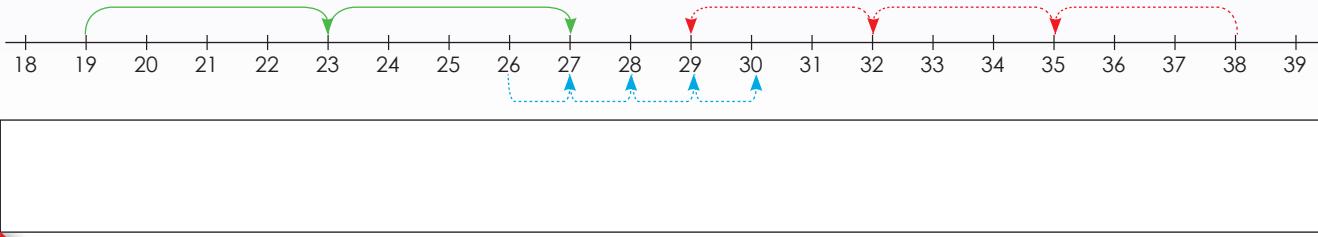
b.



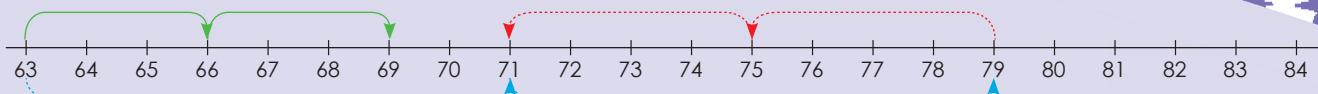
c.



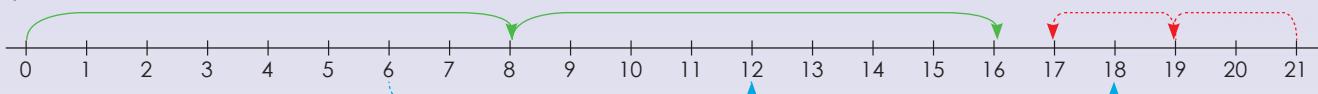
d.



e.



f.



2. Describe the rule for each pattern.

Example: 27, 36, 45, 54, 63

Rule: Adding 9 or counting in 9s

a. 6, 14, 22, 30

b. 2, 6, 10, 14, 18

c. 13, 10, 7, 4, 1

d. 8, 13, 18, 23, 28

e. 5, 9, 13, 17, 21

f. -20, -15, -10, -5, 0

g. 7, 18, 29, 40, 51

h. 1, 9, 17, 25, 33

i. 4, 5, 6, 7, 8

j. -6, -4, -2, 0, 2

Sharing

The rule is 'adding 11'. Start your pattern with 35.

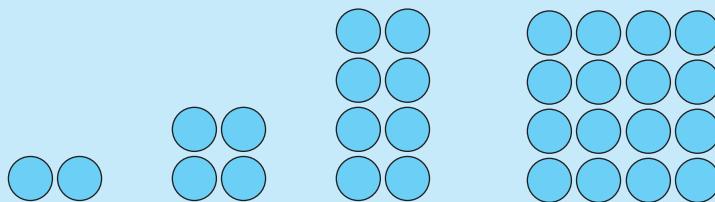
Describe the pattern.

2, 4, 8, 16, ...

$$\begin{array}{c} 2 \\ \times 2 \\ \hline 4 \\ \times 2 \\ \hline 8 \\ \times 2 \\ \hline 16 \end{array}$$

Take your time and think carefully when you identify the pattern.

Identify the **constant ratio** between consecutive terms. This pattern can be described in one's own words as "multiplying the previous number by 2".

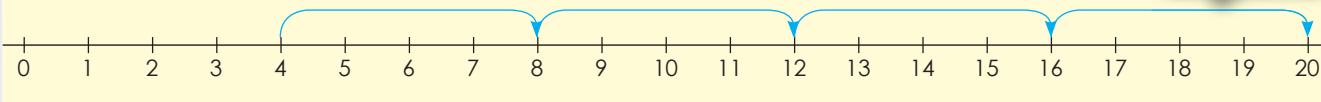


Can you still remember what constant ratio means?

1. Describe the pattern and make a number line to show each term.

Example: 4, 8, 12, 16, 20

Add 4 to the previous number.



a. 2, 8, 32, 128, 512



b. 4, 12, 36, 108, 324



c. 6, 12, 24, 48, 96



d. 8, 40, 200, 1 000, 5 000

e. 1, 6, 36, 214, 1 228

f. 3, 9, 27, 81, 243

g. 5, 20, 80, 320, 1 280

h. 7, 42, 252, 1 512

i. 9, 45, 225, 1 125

j. 10, 20, 40, 80, 160

Problem solving

If the rule is "subtracting 9", give the first five terms of the sequence starting with 104.



Numeric patterns: neither a constant difference nor a constant ratio

What is the difference between constant difference and ratio:

- constant difference, e.g. 21, 23, 25, 27, ...
- constant ratio, e.g. 2, 4, 8, 16, ...

Take your time to figure out the pattern.

Describe the pattern.

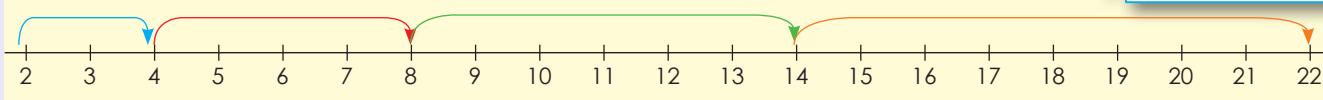
1, 2, 4, 7, 11, 16, ...

What will the next three terms be, applying the identified rule?

This pattern has neither a constant difference nor a constant ratio. It can be described as “increasing the difference between consecutive terms by one each time” or “adding one more than what was added to get the previous term”.

1. Describe the pattern and draw a number line to show each.

Example: 2, 4, 8, 14, 22



a. 8, 10, 14, 20, 28



b. 15, 12, 6, -3, -15



c. 3, 6, 10, 15, 21



d. 10, 9, 7, 4, 0



e. 6, 7, 9, 12, 21

f. 1, 3, 7, 15, 31

g. 13, 9, 4, -2, -9

h. 9, 14, 20, 27, 35

i. 24, 18, 13, 9, 7

j. 19, 20, 22, 25, 29

Problem solving

Create your own sequence without a constant ratio.



Give a rule to describe the relationship between the numbers in this sequence:
2, 4, 6, 8, ... Use the rule to find the tenth term

Position in the sequence	1	2	3	4		10
Term	2	4	6	8		?

We can represent a sequence in a table.

The "tenth term" refers to position 10 in the number sequence. You have to find a rule in order to determine the tenth term, rather than continuing the sequence up to the tenth term. You should recognise that each term in the bottom row is obtained by doubling the number in the top row. So double 10 is 20. The tenth term is 20.

1. Describe the pattern and draw a number line to show each.

Example:

Position in the sequence	1	2	3	4		10
Term	3	6	9	12		30
	1×3	2×3	3×3	4×3		10×3

a. Position in the sequence

1	2	3	4		10
Term	4	8	12	16	
	<input type="text"/>				

b. Position in the sequence

1	2	3	4		10
Term	8	16	24	32	
	<input type="text"/>				

c. Position in the sequence

1	2	3	4		10
Term	12	24	36	48	
	<input type="text"/>				

d. Position in the sequence

1	2	3	4		10
Term	7	14	21	28	
	<input type="text"/>				

e. Position in the sequence

1	2	3	4		10
Term	5	10	15	20	
	<input type="text"/>				

2. What will the term be?

Example: 5, 10, 15, 20. Position of the term $\times 5$.

Position in the sequence	1	2	3	4		15
Term	5	10	15	20		75

a.

Position in the sequence	1	2	3	4		20
Term	10	20	30	40		

b.

Position in the sequence	1	2	3	4		28
Term	3	6	9	12		

c.

Position in the sequence	1	2	3	4		35
Term	8	16	24	32		

d.

Position in the sequence	1	2	3	4		50
Term	1	8	27	64		

e.

Position in the sequence	1	2	3	4		100
Term	12	24	36	48		

f.

Position in the sequence	1	2	3	4		10
Term	15	30	45	60		

Problem solving

Thabelo is building a model house from matches. If he uses 400 matches in the first section, 550 in the second and 700 in the third section, how many matches would he need to complete the fourth section, if the pattern continued?



Number sequences and words

Look at this pattern:

4, 7, 10, 13, ...

If you consider only the relationship between consecutive terms, then you can continue the pattern ("adding 3 to previous number") up to the 20th term to find the answer. However, if you look for a relationship or rule between the term and the position of the term, you can predict the answer without continuing the pattern. Using number sequences can be useful to find the rule.

First term: $4 = 3(1) + 1$

Second term: $7 = 3(2) + 1$

Third term: $10 = 3(3) + 1$

Fourth term: $13 = 3(4) + 1$

The number in
the brackets
corresponds to the
position of the term
in the sequence.

What will the 20th pattern be?

1. Look at the following sequences:

Describe the rule in your own words.

Calculate the 20th pattern using a number sequence

Example: Number sequence: 5, 7, 9, 11

Rule in words: $2 \times$ the position of the term + 3.

20th term: $(2 \times 20) + 3 = 43$

a. **Number sequence: 2, 5, 10, 17**

Rule:

20th term:

b. **Number sequence: -8, -6, -4, -2**

Rule:

15th term:

c. **Number sequence: -1, 2, 5, 8**

Rule:

12th term:

d. **Number sequence: 6, 9, 12, 15**

Rule:

19th term:

e. **Number sequence:** -6, -2, 2, 6

Rule:

18th term:

f. **Number sequence:** 7, 12, 17, 22

Rule:

12th term:

g. **Number sequence:** 2,5; 3; 3,5; 4

Rule:

21st term:

h. **Number sequence:** -3, -1, 1, 3

Rule:

15th term:

i. **Number sequence:** 3, 7, 11, 15

Rule:

14th term:

j. **Number sequence:** 14, 24, 34, 44

Rule:

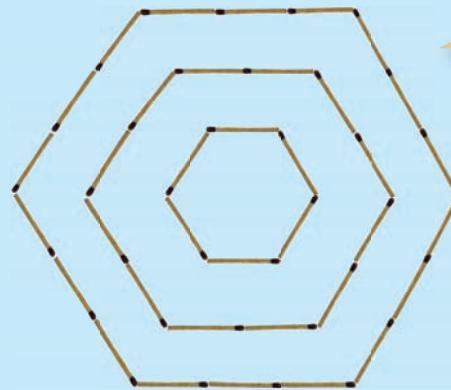
25th term:

Problem solving

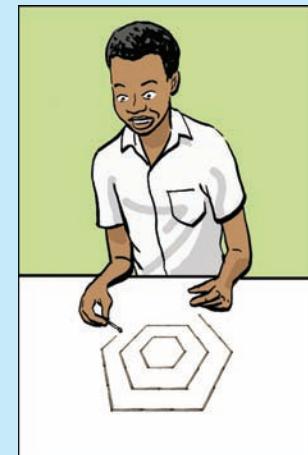
Miriam collects stickers for her sticker album. If she collects 4 stickers on day 1, 8 on day 2, 16 on day 3 and 32 on day 4, how many would she collect on day 5 if the pattern continued?

Helen spends 2 hours playing computer games on the first day of the school holidays. On the second day she plays for 5 hours and on the third day she plays for 8 hours. For how many hours would she play on the fourth day if she kept on playing?

What do you see? Describe the pattern



Take your time to explore the pattern.



1. Create the first three terms of the following patterns with matchsticks and then draw the patterns in your book. Complete the tables.

a. Triangular pattern

Position of a square in pattern	1	2	3	4	5	6	7
Number of matches							

b. Square pattern

Position of a square in pattern	1	2	3	4	5	6	7
Number of matches							

Position of a square in pattern	1	2	3	4	5	6	7
Number of matches							

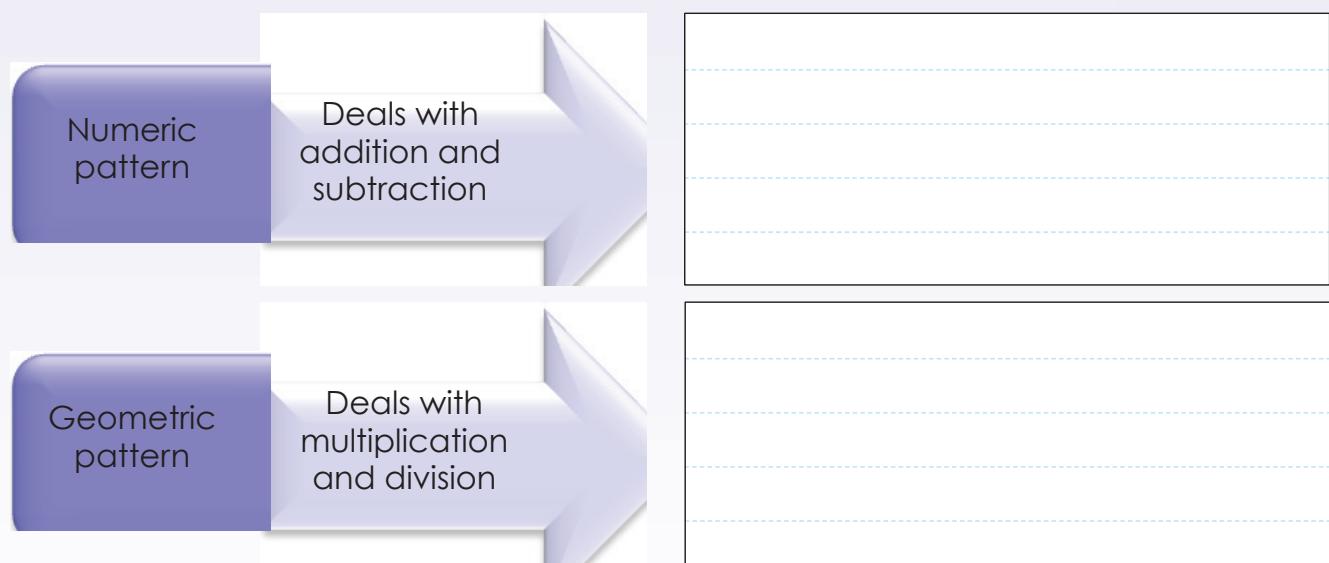
c. Rectangular pattern

Position of a square in pattern	1	2	3	4	5	6	7
Number of matches							

d. Pentagonal pattern

Position of a square in pattern	1	2	3	4	5	6	7
Number of matches							

2. Look at worksheets 81-86 again. Explain and give examples of the following:

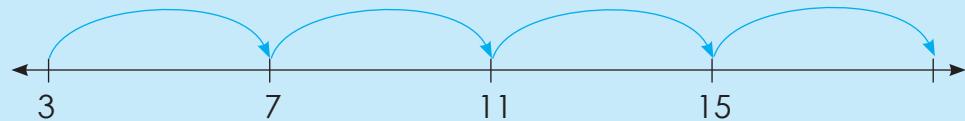


Problem solving

Represent an octagonal number pattern.

Look at the example and describe it.

Adding 4 to the previous term



4 times the position of the term - 1

Position in the sequence	1	2	3	4
Term	3	7	11	15

$1(n) - 1$, where n is the position of the term.

$$1 \times 4 - 1$$

$$2 \times 4 - 1$$

$$3 \times 4 - 1$$

$$4 \times 4 - 1$$

First term: $3 = 4(1) - 1$

Second term: $7 = 4(2) - 1$

Third term: $11 = 4(3) - 1$

Fourth term: $15 = 4(4) - 1$

1. Describe the sequence in different ways using the template provided.

a. 5, 11, 17, 23

i)

ii)

Position in the sequence	1	2	3	4
Term				

iii)

, where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

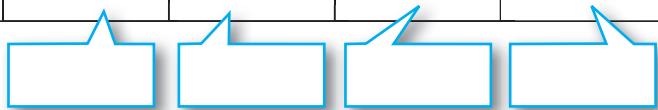
b. 5, 7, 9, 11 ...

i)



ii)

Position in the sequence	1	2	3	4
Term				



iii)

, where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

c. 10, 19, 28, 37, ...

i)



ii)

Position in the sequence	1	2	3	4
Term				



iii)

, where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

continued 

Numeric patterns: describe a pattern continued

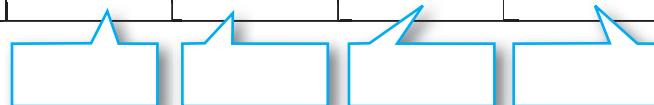
d. 0, 4, 8, 12, ...

i)



ii)

Position in the sequence	1	2	3	4
Term	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>



iii)

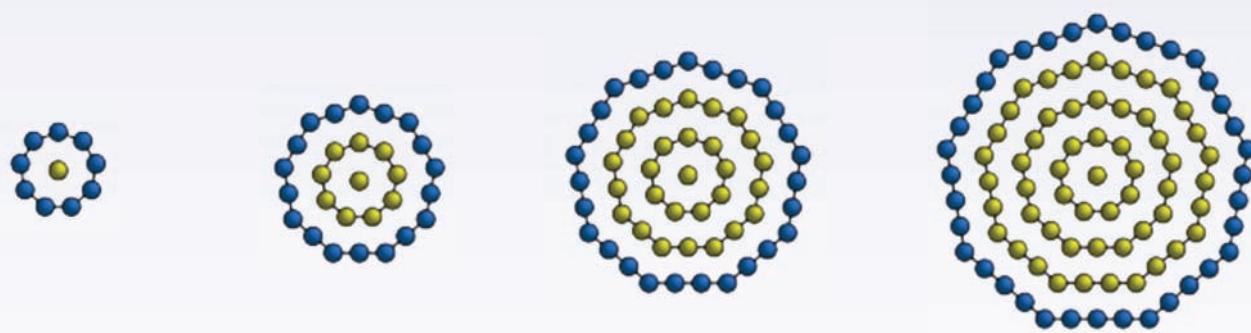
, where n is the position of the term.

First term:

Second term:

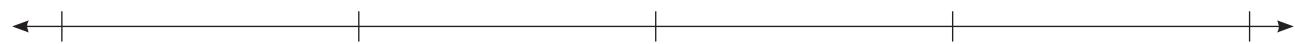
Third term:

Fourth term:



e. 14, 25, 36, 47 ...

i)



ii)

Position in the sequence	1	2	3	4
Term	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

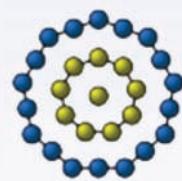
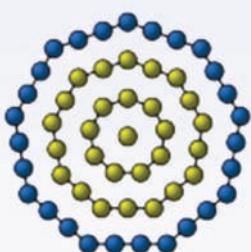
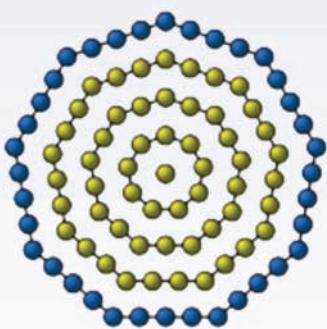
iii) , where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:



Problem solving

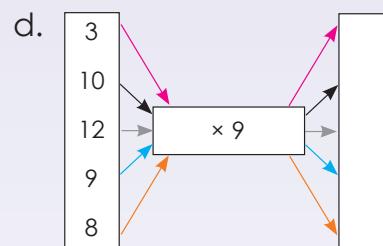
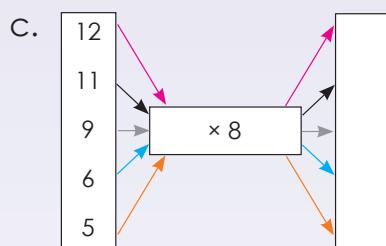
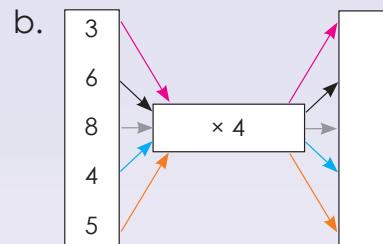
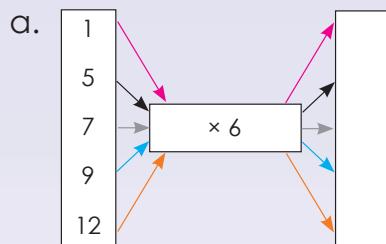
What is the 30th term if n is the n^{th} position in $8(n) - 7$?



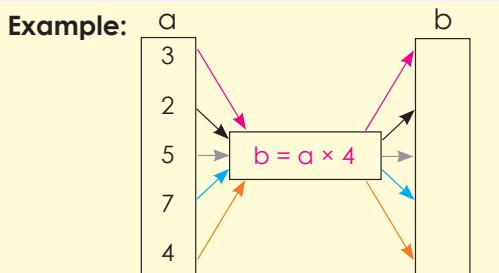
What does input and output mean? Make a drawing to show a real life example.



1. Complete the flow diagrams.

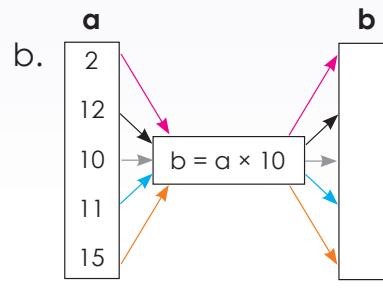
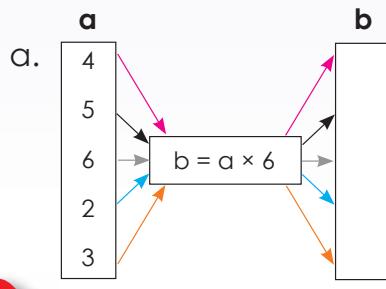


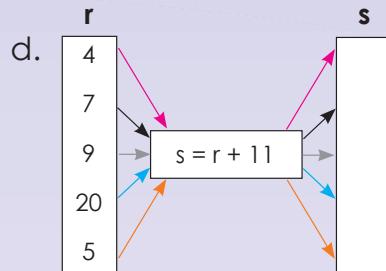
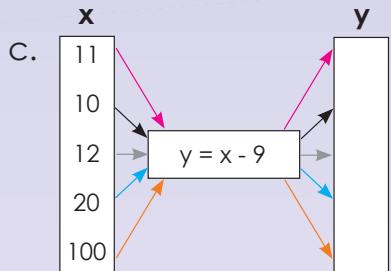
2. Use the given rule to calculate the value of b.



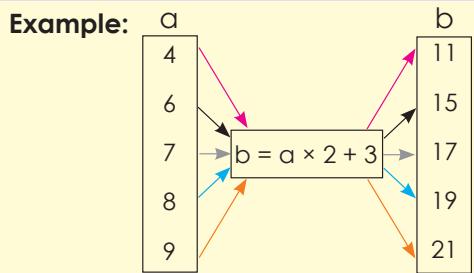
$b = a \times 4$

- $3 \times 4 = 12$
- $2 \times 4 = 8$
- $5 \times 4 = 20$
- $7 \times 4 = 28$
- $4 \times 4 = 16$

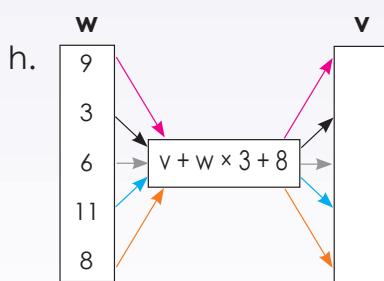
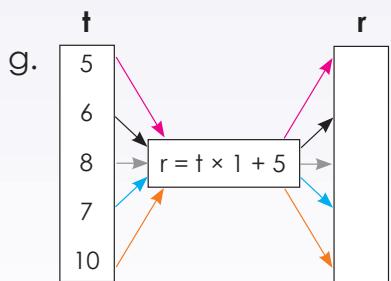
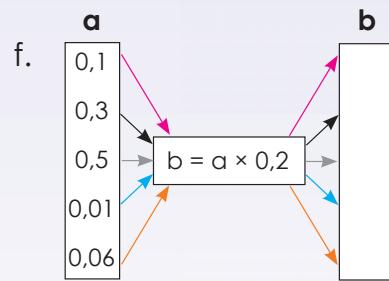
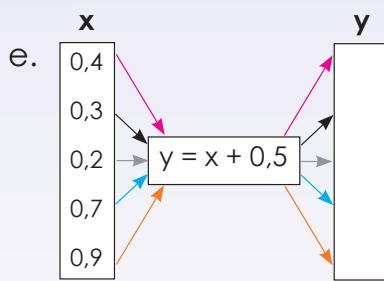
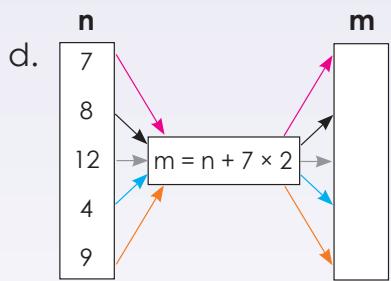
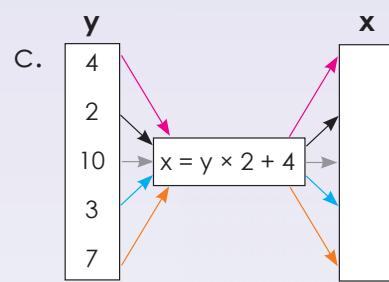
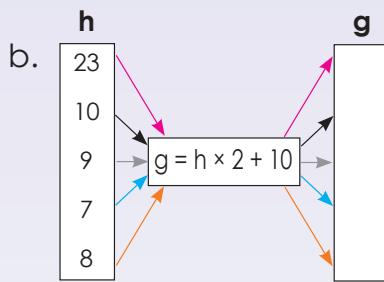
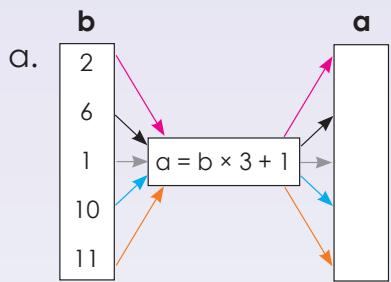




3. Use the given rule to calculate the variable.



- $4 \times 3 + 3 = 11$
- $6 \times 3 + 3 = 15$
- $7 \times 2 + 3 = 17$
- $8 \times 2 + 3 = 19$
- $9 \times 2 + 3 = 21$



Problem solving

Draw your own spider diagram where $a = b + 7$.

Draw your own spider diagram where $a = b \times 2 + 11$

Functions and relationships

Discuss this:

The rule is $y = x + 5$

x	1	2	3	10	100
y	6	7	8	15	105

$$y = 1 + 5 \\ = 6$$

$$y = 2 + 5 \\ = 7$$

$$y = 3 + 5 \\ = 8$$

$$y = 10 + 5 \\ = 15$$

$$y = 100 + 5 \\ = 105$$

1. Complete the table below

Example: See introduction

a. $x = y + 2$

x	2	4	6	8	10	20
y						

b. $a = b + 7$

b	1	2	3	4	5	10
a						

c. $m = n + 4$

n	3	4	5	6	7	10	100
m							

d. $x = z \times 2$

z	2	3	4	5	6	7
x						

e. $y = 2y - 2$

x	1	2	3	4	5	6	7
y							

f. $m = 3n + 2$

n	1	5	10	20	25	100
m						

1. What is the value of m and n ?

Example:

x	1	2	3	4		18	m	51
y	8	9	10	11		25	39	n

$$y = x + 7$$

$$y = 51 + 7$$

$$y = 58$$

$$\therefore n = 58$$

$$y = x + 7$$

$$39 = x + 7$$

$$39 - 7 = x + 7 - 7$$

$$32 = x$$

$$\therefore m = 32$$

Rule: the given term plus 7

$n = 58$ and $m = 32$

a.

x	1	2	3	4		25	m	51
y	10	11	11	13		n	39	60

 m n

b.

x	1	2	3	4		m	30	60
y	2	4	6	8		22	n	120

 m n

c.

x	1	2	3	4		10	15	m
y	5	10	15	20		50	n	90

 m n

d.

x	1	2	3	4		7	m	46
y	13	14	15	16		19	24	n

 m n

e.

x	1	2	3	4		6	10	m
y	3	6	9	12		18	n	60

 m n

Problem solving

- What is the tenth pattern? ($3 \times 7, 4 \times 7, 5 \times 7, \dots$)
- If ($x = 2y + 9$ and $y = 2, 3, 4, 5, 6$), draw a table to show it.



Algebraic expressions and equations

Compare the two examples.

$5 + 4$

$5 + 4 = 9$

What do you notice?

What is on the left-hand side of the equal sign?

The left-hand side is an **expression**, $5 + 4$, that is equal to the value on the right-hand side, 9.

What is on the right-hand side?

$5 + 4 = 9$ is called an **equation**. The left-hand side of an equation is equal to the right-hand side.

An equation is a mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

1. Say if it is an expression or an equation.

Example: $8 + 3$ (It is an expression)

$8 + 3 = 11$ (It is an equation)

a. $4 + 8$

b. $9 + 7 = 16$

c. $7 + 6 =$

d. $3 + 5 = 8$

e. $11 + 2 =$

f. $9 + 7 =$

2. Describe the following.

Example: $6 + 2 = 8$

This is an **expression**, $6 + 2$, that is equal to the value on the right-hand side, 8.

$6 + 2 = 8$ is called an **equation**. The left-hand side of an equation equals the right-hand side.

a. $9 + 1 = 10$

b. $3 + 5 = 8$

c. $9 = 5 + 4$

d. $7 = 1 + 6$

e. $11 = 5 + 6$

f. $8 + 9 = 17$

3. Make use of the variable “a” to create 3 expressions of your own.

Example: $5 + a = 13$

4. Say if it is an expression or an equation.

Example: $8 + a$ (It is an expression)

$8 + a = 11$ (It is an equation)

a. $5 + a =$

b. $6 + a = 12$

c. $7 + b = 8$

d. $8 + b =$

e. $9 + a = 18$

f. $6 + b =$

5. What would “a” be in question 4. a, b, and e? _____

6. What would “b” be in question 4. c, d and f? _____

Problem solving

Write an equation for the following. I have 12 sweets. In total Phelo and I have 18 sweets. How many sweets does Phelo have?



Algebraic expressions

1, 3, 5, 7, 9 ...

Describe the rule of this number sequence in **words**.

Adding 2 to the previous term.

What does the rule $2n - 1$ mean for the number sequence 1, 3, 5, 7, 9, ...?

Position in sequence	1	2	3	4	5	n
Term	1	3	5	7	9	
1 st term: $2(1) - 1$	2 nd term: $2(2) - 1$	3 rd term: $2(3) - 1$	4 th term: $2(4) - 1$	5 th term: $2(5) - 1$	n^{th} term: $2(n) - 1$	

What is the rule as an **expression**? $2(n) - 1$

1. Describe the following in words.

Example: 4, 8, 12, 16, 20, ...

Adding 4 to the previous pattern

a. 3; 6; 9; 12; ...

b. 10; 20; 30; 40; ...

c. 7; 14; 21; 28; ...

d. 6; 12; 18; 24; ...

e. 8; 16; 24; 32; ...

f. 5; 10; 15; 20; ...

2. Describe the following sequence using an expression.

Example: 4, 8, 12, 16, 20, ...

Position in sequence	1	2	3	4	5	n
Term	4	8	12	16	20	

First term: $3(1) + 1$

a. 6; 11; 16; 21; ...

Position in sequence	1	2	3	4	5	n
Term						

b. 3; 5; 7; 9; 11; ...

Position in sequence	1	2	3	4	5	n
Term						

c. 9; 15; 21; 27; ...

Position in sequence	1	2	3	4	5	n
Term						

3. What does the rule mean?

Example: the rule $2n - 1$ means for the following number sequence: 1, 3, 5, 7, 9 ...

Position in sequence	1	2	3	4	5	n
Term	1	3	5	7	9	

a. The rule $3n - 1$ = means for the following number sequence

Position in sequence						
Term						

b. The rule $4n - 3$ = means for the following number sequence

Position in sequence						
Term						

c. The rule $6n - 2$ = means for the following number sequence

Position in sequence						
Term						

d. The rule $5n - 5$ = means for the following number sequence

Position in sequence						
Term						

e. The rule $7n - 4$ = means for the following number sequence

Position in sequence						
Term						

Problem solving

Write an algebraic expression for the following: Sipho built 3 times more puzzles than I did last holiday.

Describe the rule of this number sequence in **words**.

5, 9, 13, 17, 21, ...

Adding 2 to the previous term.

What does the rule $4n + 1$ mean for the number sequence 5, 9, 13, 17, 21, ...

First term: $4(1) + 1$
 Second term: $4(2) + 1$
 Third term: $4(3) + 1$
 Fourth term: $4(4) + 1$
 Fifth term: $4(5) + 1$
 n^{th} term: $4(n) + 1$

The rule as an **expression**

1. Describe the following in words.

Example: 2, 6 10, 14, 18, ...

Adding 4 to the previous number.

a. 3; 5; 7; 9; ...

b. 5; 10; 15; 20; ...

c. 21; 18; 15; 12; ...

d. 99; 98; 97; 96; ...

e. 4; 8; 12; 16; ...

f. 7; 14; 21; 28; ...

2. Describe the following sequence using an expression.

Example: 2, 6, 10, 14, 18, ...

First term: $4(1) - 2$

a. 2; 4; 5; 6; 10; ...

b. 3; 5; 7; 9; 11; ...

c. 8; 16; 24; 32; ...

d. 5; 10; 15; 20; ...

3. If the rule is ___, what could the sequence be?
Create five possible answers for each.

a. "Adding 7"

b. "Subtracting 9"

c. "Adding 5"

d. "Subtracting 8"

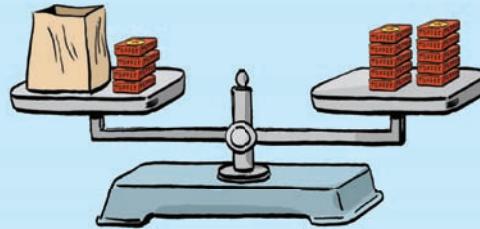
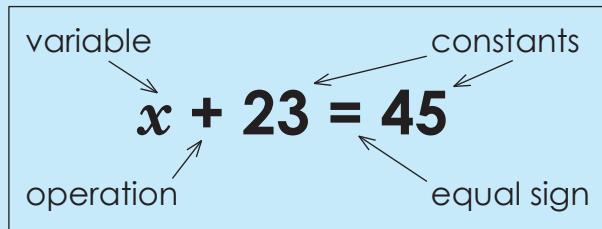
e. "Adding 3" "Subtracting 4"

Problem solving

If the rule is "adding $\frac{1}{4}$ ", what could the sequence be? Create five possible answers.



Look at and describe:



Read and answer:

Imagine that on the right-hand side of this balance scale there are 10 equal mass objects and on the left-hand side there are 4 similar objects and an unknown number of other objects in a bag. The scale is balanced; therefore, we know that there must be an equal mass on each side of the scale.

Explain how you would find out how many objects there are in the bag.

1. Solve x.

Example: $x + 5 = 9$

$$x + 5 - 5 = 9 - 5$$

$$x = 4$$

a. $x + 12 = 30$

b. $x + 8 = 14$

c. $x + 17 = 38$

d. $x + 20 = 55$

e. $x + 25 = 30$

f. $x + 18 = 26$

2. Solve for x.

Example: $x - 5 = 2$

$$x - 5 + 5 = 2 + 5$$

$$x = 7$$

a. $x - 7 = 5$

b. $x - 3 = 1$

c. $x - 15 = 12$

d. $x - 17 = 15$

e. $x - 23 = 20$

f. $x - 28 = 13$

3. Solve for x.

Example: $x + 4 = -7$

$$x + 4 - 4 = -7 - 4$$

$$x = -11$$

a. $x + 3 = -15$

b. $x + 7 = -12$

c. $x + 2 = -5$

d. $x + 5 = -15$

e. $x + 12 = -20$

f. $x + 10 = -25$

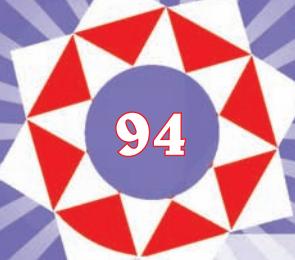
Problem Solving

Write an equation for the following and solve it.

Jason read 7 books and Gugu read 11 books. How many books did they read altogether?

Rebecca and her friend read 29 books altogether. Rebecca read 14 books. How many books did her friend read?

Bongani buys 12 new CDs and Sizwe buys 14. How many CDs did they buy together.



More algebraic equations

$$2x = 30$$

What does $2x$ mean?

($2x$ means 2 multiplied by x)

What is the inverse operation of multiplication?

Division

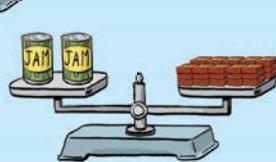
We need to divide $2x$ by 2 to solve for x .

$$\frac{2x}{2} = \frac{30}{2}$$

$$x = 15$$

Remember you need to keep the two sides of the equation balanced. What you do on the one side of the equal sign, you must do on the other side as well.

$$2x = 30$$



1. Solve for x.

Example: $3x = 12$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

a. $5x = 20$

b. $2x = 8$

c. $2x = 18$

d. $4x = 48$

e. $3x = 27$

f. $5x = 30$

g. $10x = 100$

h. $9x = 81$

i. $15x = 45$

j. $7x = 14$

2. Solve for x.

Example: $3x - 2 = 10$

$$3x - 2 + 2 = 10 + 2$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

a. $7x - 2 = 12$

b. $4x - 4 = 12$

c. $3x - 1 = 2$

d. $2x - 1 = 7$

e. $5x - 3 = 17$

f. $5x - 7 = 13$

g. $6x - 5 = 25$

h. $9x - 8 = 82$

i. $8x - 7 = 49$

j. $3x - 2 = 16$

Problem Solving

Create an equation and solve it. How fast can you do it?

Two times y equals sixteen.

Five times c equals sixty three.

Eight times x equals sixteen.

Sixteen times b equals four.

Eight times t equals eighty.

Three times d equals thirty nine.

Nine times q equals eighty one.

Five times y equals hundred.

Seven times a equals twenty one.



What do the following equations mean?

$$P = 4l$$

The perimeter of a square is 4 times the length.

$$P = 2l + 2b$$

The perimeter of a rectangle is 2 times the length plus 2 times the breadth.

$$A = l^2$$

The area of a square is the length squared.

$$A = l \times b$$

The area of a rectangle is length times breadth.

Note that you did perimeter and area in the previous terms

1. Solve for x.

Example: If $y = x^2 + 2$, calculate y when $x = 4$

$$y = 4^2 + 2$$

$$y = 16 + 2$$

$$y = 18$$

a. $y = x^2 + 2; x = 4$

b. $y = b^2 + 10; b = 1$

c. $y = a^2 + 4; a = 4$

d. $y = r^2 + 3; r = 5$

e. $y = p^2 + 7; p = 6$

f. $y = c^2 + 7; c = 7$

2. Calculate the following:

Example: What is the perimeter of a rectangle if the length is 2cm and the breadth is 1,5cm?

$$P = 2l + 2b$$

$$P = 2(2\text{cm}) + 2(1,5 \text{ cm})$$

$$P = 4\text{cm} + 3\text{cm}$$

$$P = 7\text{cm}$$

a. The perimeter of a rectangle where the breadth equals 2,2 cm and the length equals 2,5 cm

b. The area of a square if the breadth equals 3,5 cm.

c. The perimeter of a square if the breadth equals 4,2 cm.

d. The area of a rectangle if the length is 3,5 cm and breadth is 2,5 cm

e. The area of a square if the length is 5 cm.

f. The perimeter of a rectangle if the breadth is 4,3 cm and length is 8,2 cm.

g. The perimeter of a square if the length is 2,6 cm.

h. The perimeter of a rectangle if the breadth is 8,5 cm and the length is 12,4 cm.

i. The area of a rectangle if the breadth is 10,5 cm and length is 15,5 cm.

h. The perimeter of a rectangle if the breadth is 3,5 cm and the length is 6,7 cm

Problem Solving

Write an equation and then solve it for each of these.

What is the perimeter of the swimming pool if the breadth is 12 m and the length is 16 m.

Work out the area of a square if the one side is equal to 5,2 cm.

What is the perimeter of a rectangle if the length is 5,1 cm and the breadth is 4,9 cm.

Establish the area of your bedroom floor for new tiles the length is 4,5 m and the breadth is 2,8 m.

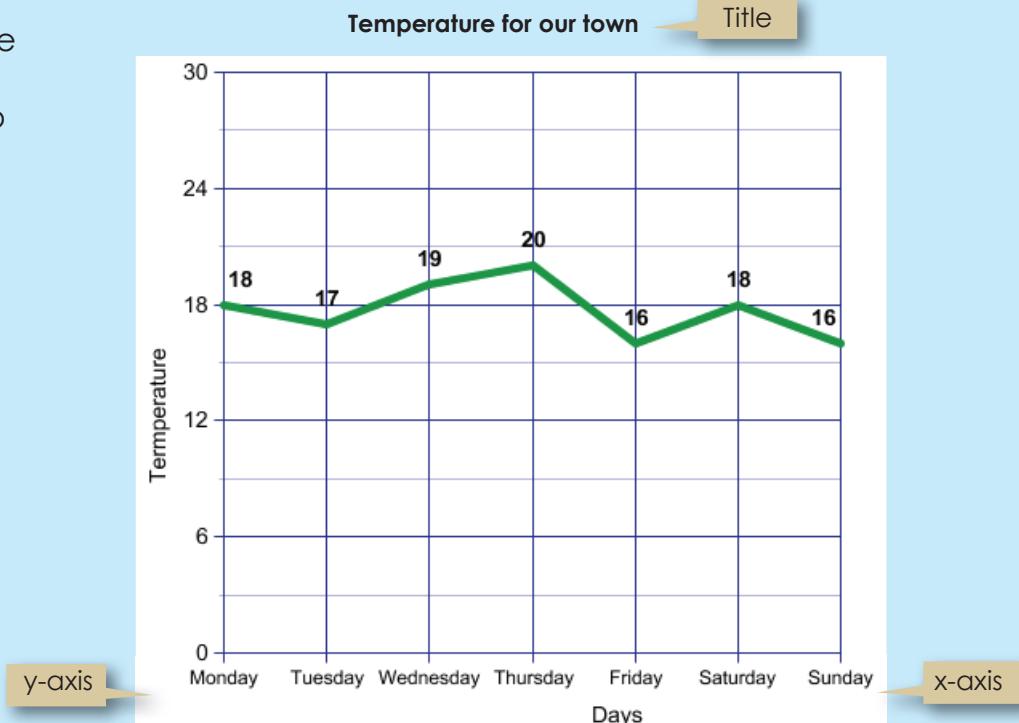




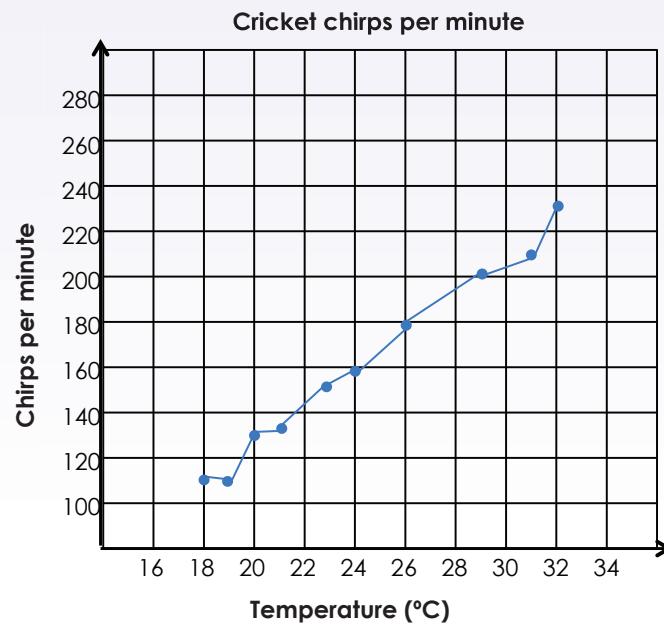
Interpreting graphs: temperature and time graphs

Look at the graph and talk about it.

Would you make any changes or add anything to the graph?



1. Thebogo heard that nature lovers use the chirping of crickets to estimate the temperature. The last time he went camping he brought a thermometer so he could collect the data on the number of cricket chirps per minute for various temperatures. The first thing Thebogo did was make the graph below.



a. What is the temperature if the cricket chirps:

- i. 120 times? _____
- ii. 150 times? _____
- iii. 160 times? _____
- iv. 230 times? _____
- v. 270 times? _____



b. Thebogo counts 190 cricket chirps in a minute. What would the temperature be?

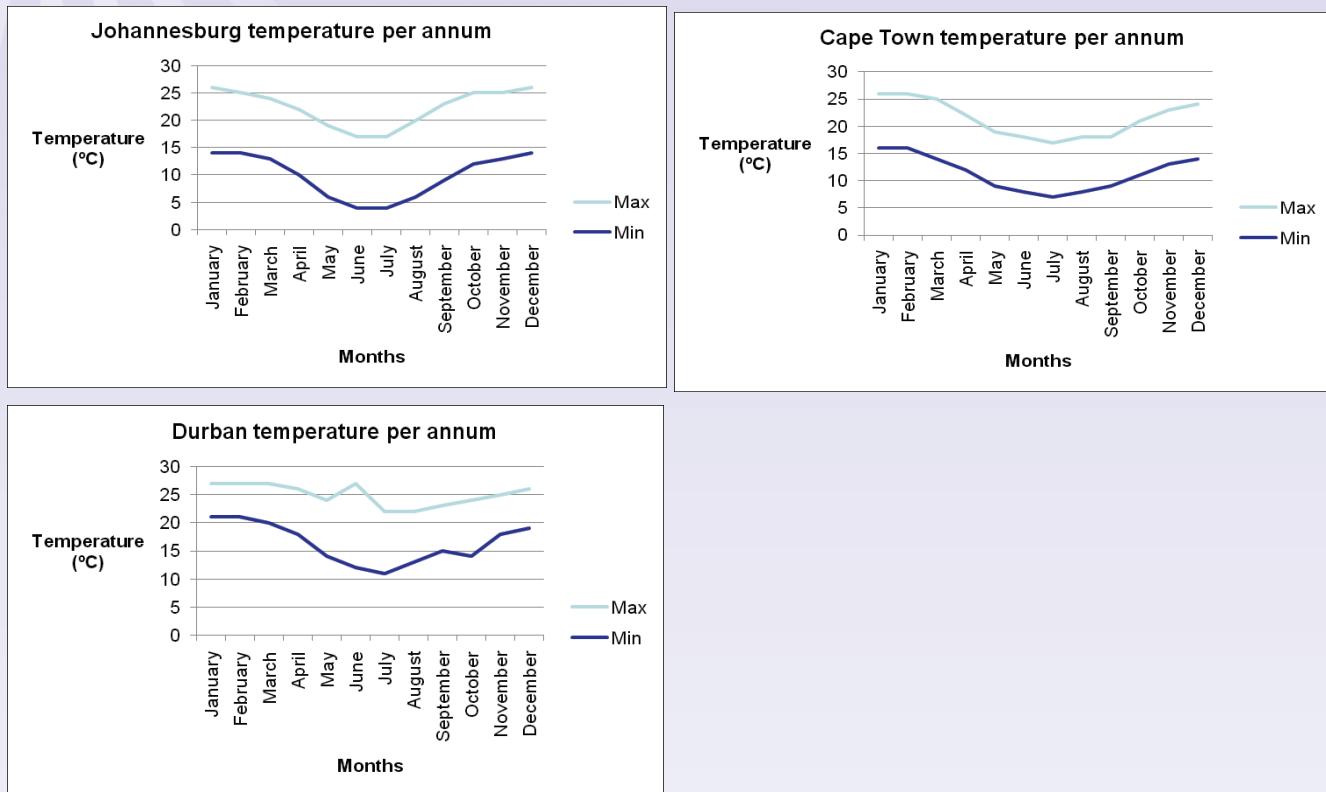
c. Thebogo notices that the number of cricket chirps per minute drops by 30 chirps per minute. What could she conclude about the change in temperature?

d. Use the words increasing and decreasing to describe the graph.



Interpreting graphs: temperature and time graphs continued

2. Average temperature per annum for Johannesburg, Cape Town and Durban.



a. What is the average maximum temperature for:

- Durban in August _____
- Cape Town in July _____
- Johannesburg in April _____
- Durban in July _____
- Cape Town in September _____

b. What is the average minimum temperature for:

- Johannesburg in April _____
- Cape Town in October _____
- Johannesburg in September _____
- Durban in March _____
- Cape Town in July _____

c. What is the difference in maximum temperature between:

- i. Durban and Johannesburg in April _____
- ii. Cape Town and Durban in October _____
- iii. Johannesburg and Cape Town in May _____
- iv. Durban and Johannesburg in September _____
- v. Cape Town and Johannesburg in April _____

d. Describe the graphs using the words increasing and decreasing.

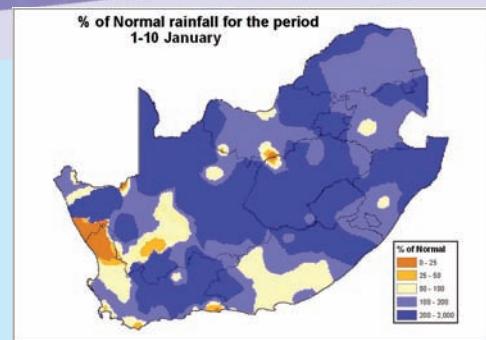
Problem solving

What is the difference between the minimum and maximum temperatures of Durban, Cape Town and Johannesburg in December? Which province would you most like to visit in December. Why?

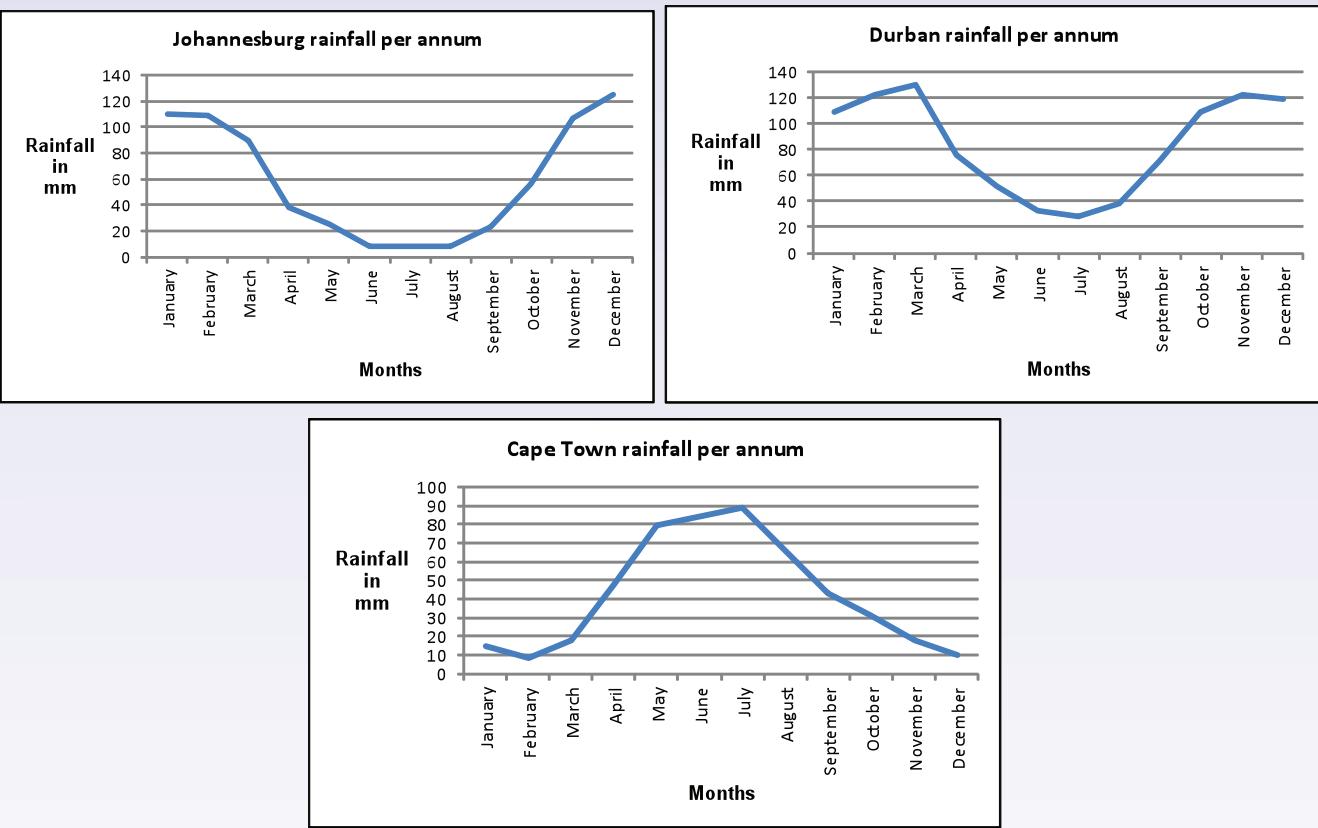
Interpreting graphs: rainfall and time graphs

Look at the graphs and answer:

- What does each graph represent?
- What is the heading of each graph?
- What is the x-axis telling us?
- What is the y-axis telling us?



1. Look at the graphs and answer the following questions:



a. What is the heading of each graph?

b. What is the x-axis telling us?

c. What is the y-axis telling us?

d. Which province's average rainfall is the highest in October?

e. Which province's average rainfall is the lowest in April?

f. Which province will you visit in December? Why?

g. Which province will you not visit in December? Why?

--

h. Which province(s) have a winter rainy season? Why do you say so?

--

i. Which province(s) have a summer rainy season? Why do you say so?

1. **What is the primary purpose of the study?**

j. Use the words increasing and decreasing to describe each graph.

1. **What is the primary purpose of the study?**

2. Use the graphs to complete the following tables.

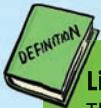
What is our weather?

What is the highest rainfall per year for your town? Which month? Keep a record during a rainy month and draw a graph.

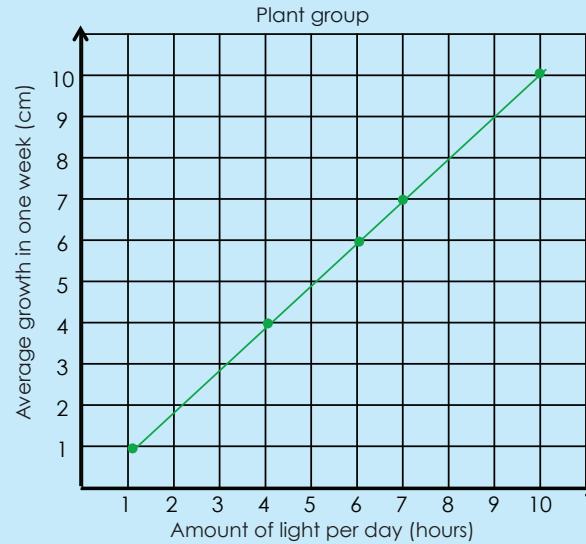


Sam kept this record of plants growing. Discuss.

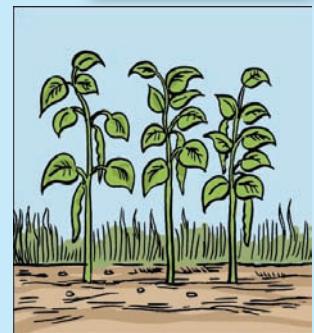
Would you make any changes or add anything to the graph?



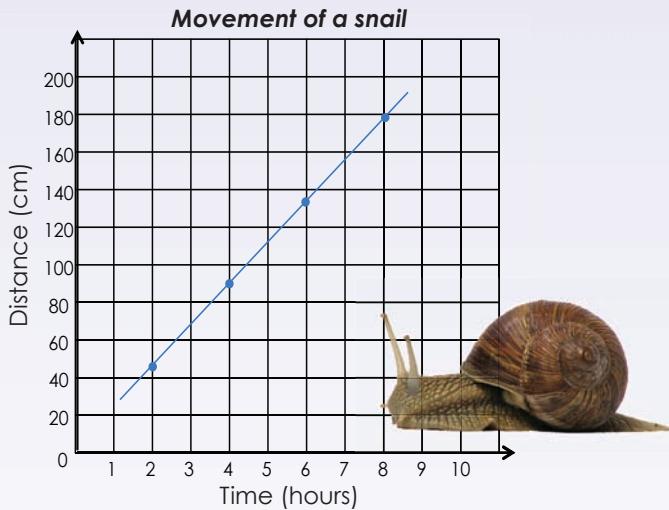
Linear equation:
The graph from a linear equation is a straight line.



Is this graph:
decreasing or
increasing?



1. Answer the following questions on the movement of a snail.



d. How far will a snail move in two hours?
How did you use the graph to work this out?

f. Why is this a linear graph?

a. How far will a snail move in eight hours?

b. How far will a snail move in four hours?
How did you use the graph to work this out?

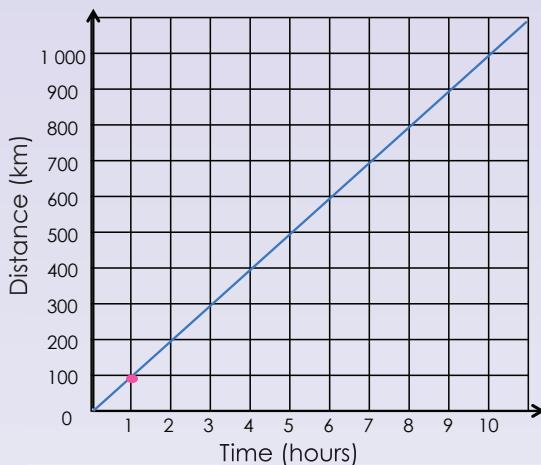
c. How far will a snail move in six hours?
How did you use the graph to work this out?

e. How far will a snail move in 9 hours?
How did you use the graph to work this out? Plot this on the graph.

g. Is this graph increasing or decreasing?

2. The graph below shows the distances travelled by car from Gauteng to Cape Town.

Travelling from Gauteng to Cape Town



How long did it take the person to travel _____ km? Show the co-ordinate on the graph and explain it. We did the first one for you.

Example: 900 km

It took the person nine hours to travel 900 km.

We can write it as (nine hours, 900 km).

a. 100 km

b. 500 km

c. 800 km

d. 750 km

e. 300 km

f. 250 km

3. How far did the person travel in:

a. 1 hour

b. 1 hour 30 minutes

c. 3 hours

d. 4 hours 30 minutes

e. 5 hours

f. 2 hours 30 minutes

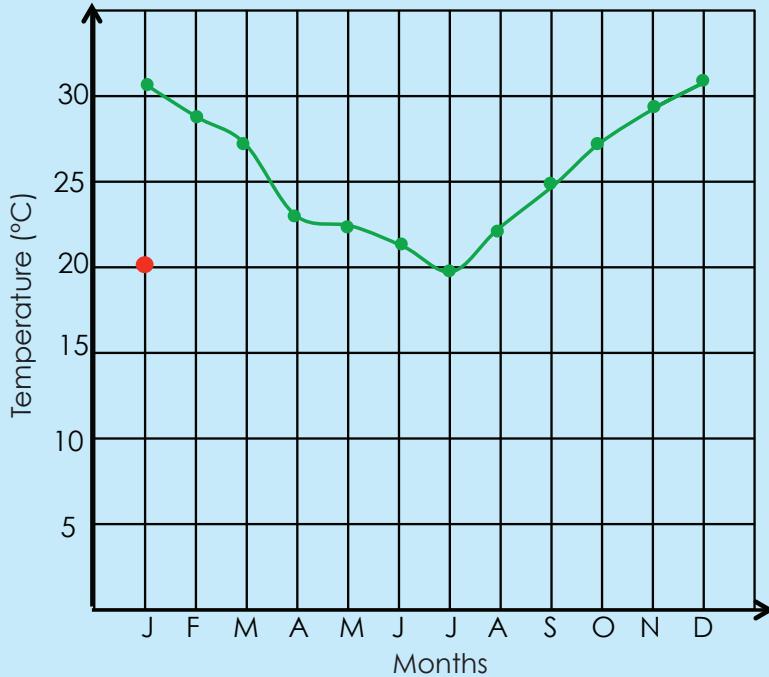
What is our weather?

Use the graph on "Travelling from Gauteng to Cape Town" to work out how long it will take to travel 275 km?



You kept this record but forgot to plot the minimum temperature. Plot it using the information from your notes.

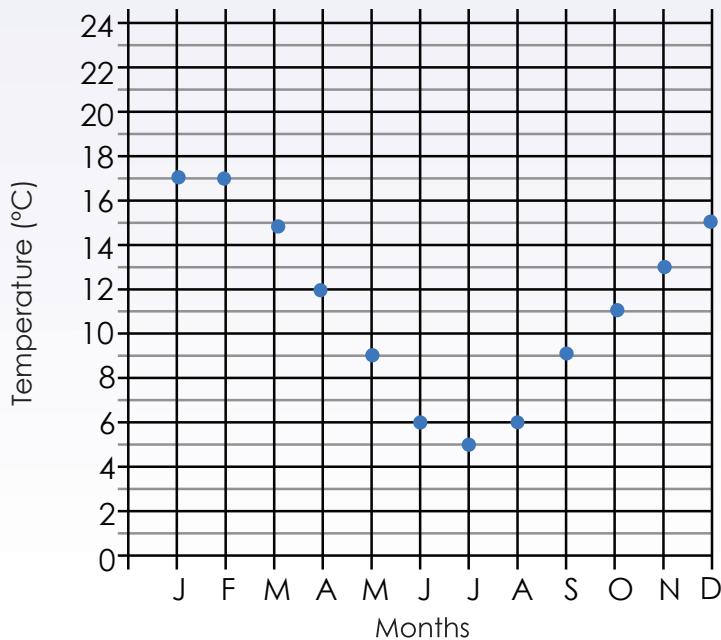
Average maximum temperature for our town



January: 20°C
February: 19°C
March: 15°C
April: 12°C
May: 10°C
June: 5°C
July: 4°C
August: 6°C
September: 9°C
October: 12°C
November: 15°C
December: 18°C

1. Answer the questions on the graph.

Average minimum temperature for our town



a. What is the heading of the graph?

b. What is the scale on the x-axis?

c. What is the scale on the y-axis?

d. What is the x-axis telling us?

e. What is the y-axis telling us?

f. What are the points or dots telling us?



continued ➔



Drawing graphs continued

2. Use the grid paper on the next page to draw a graph for this table.

Month	Maximum	Minimum
J	30	16
F	29	17
M	28	14
A	26	12
M	24	8
J	21	6
J	21	5
A	22	6
S	24	8
O	25	12
N	26	13
D	28	15

Use the entire sheet to draw your graph.

You should determine your intervals carefully.



a. What will be written on your x-axis?

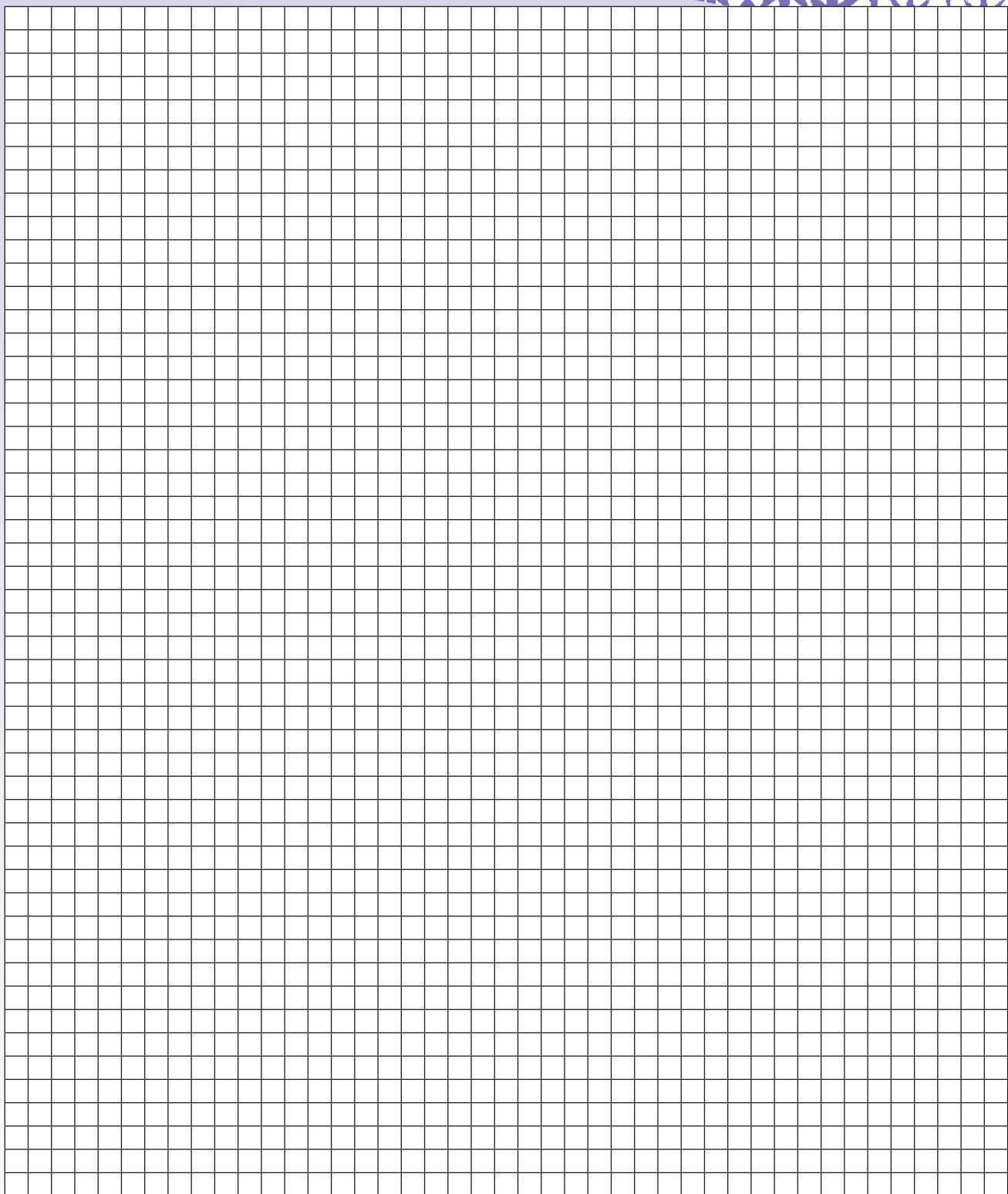
c. What will the scale of the y-axis be?

e. What will your graph show?

b. What will be written on your y-axis?

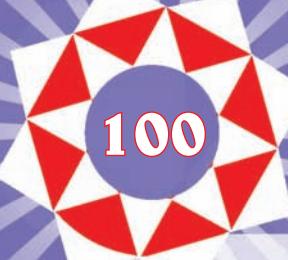
d. What will the heading of your graph be?

f. Describe the graph using the following words: increasing, decreasing, linear and non-linear.



Research

Draw a graph showing the maximum and minimum temperatures annually for any other country than South Africa.



Drawing graphs

You have to draw a graphs with the following values. How will you do it?

The maximum value of the y-axis is 24.

The maximum value of the x-axis is 60.

The scale could be:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

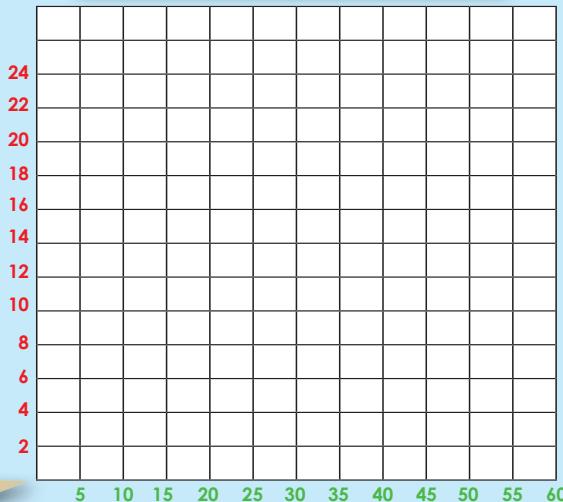


Why are these intervals in 2s and not in 1s or 3s.

Why are these intervals in 5s and not in 2s or 10s?

The scale could be:

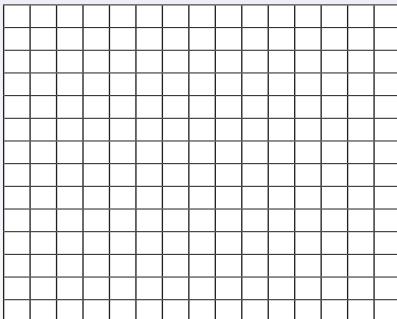
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24



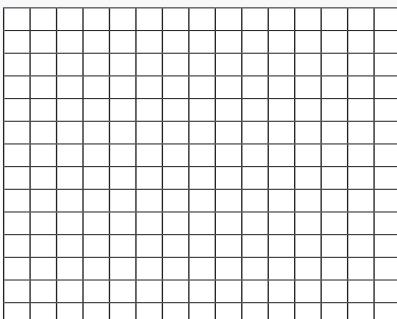
1. In this activity you should use the grid paper to draw your graph. Determine the scale for the y-axis and x-axis.

The maximum value of:

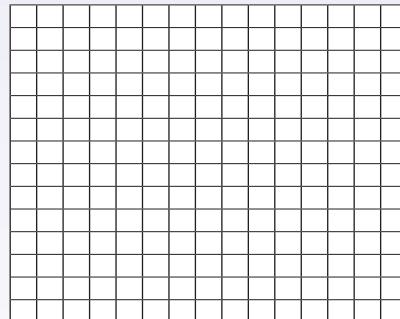
a. x-axis is 45 and y-axis is 24



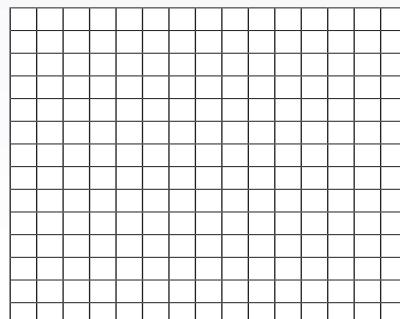
c. x-axis is 40 and y-axis is 30



b. x-axis is 75 and y-axis is 72

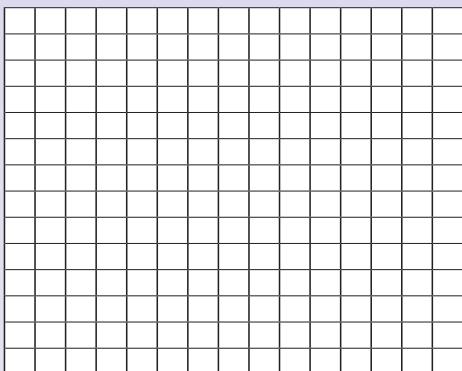


d. x-axis is 100 and y-axis is 100

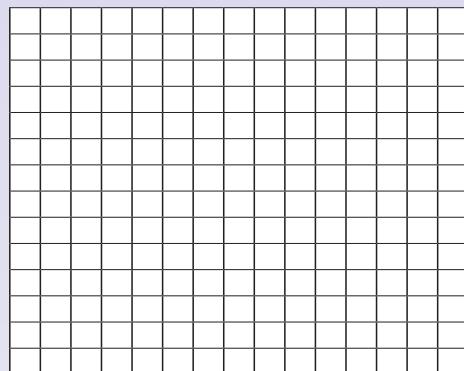


2. Draw the scales for the following graphs.

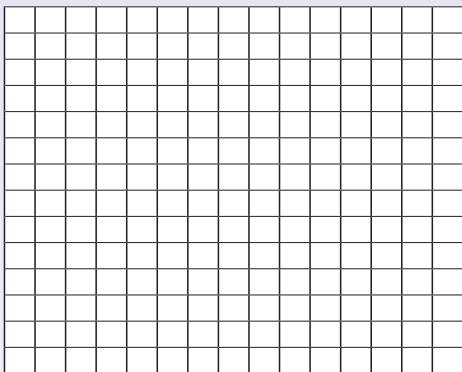
a. x-axis: 0, 3, 6, 9, 12, 15 and
y-axis: 0, 5, 10, 15, 20, 25, 30



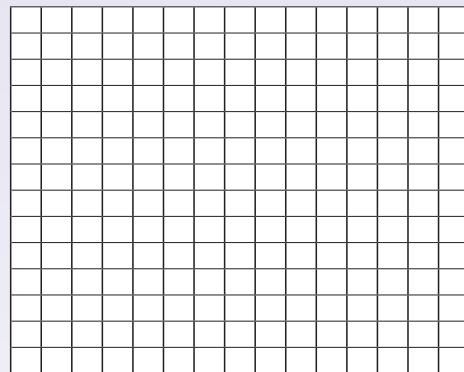
b. x-axis: 0, 4, 8, 12 and
y-axis: 0, 10, 20, 30, 40, 50, 60



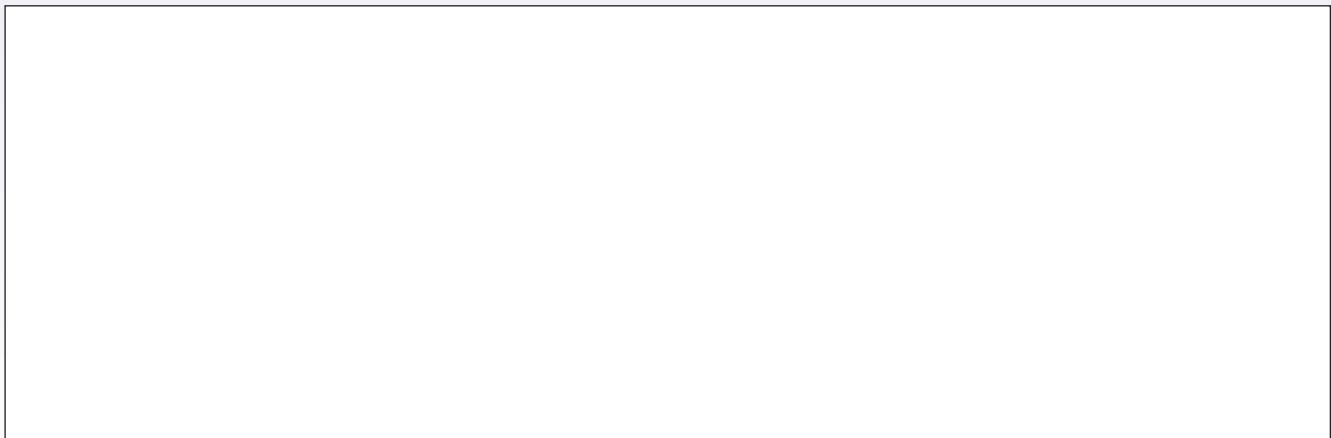
c. x-axis: 0, 5, 10, 15, 20, 25, 30, 35, 40 and
y-axis: 0, 20, 40, 60, 80, 100



d. x-axis: 36, 48, 60, 72, 84 and
y-axis: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20



3. Cut and paste a graph from a newspaper. Describe the intervals.

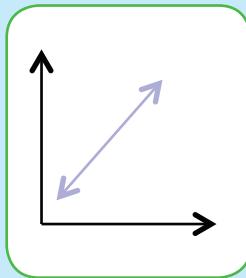


Drawing graphs

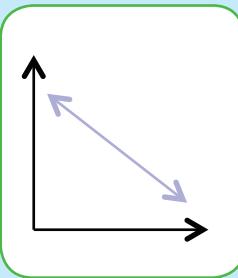
Draw a graph with 10 intervals on the x-axis and 12 intervals on the y-axis. You can use any multiple to label it.



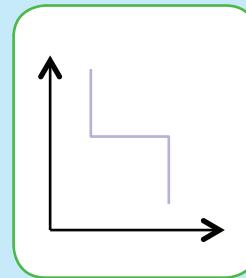
Look at the graphs. Explain them.



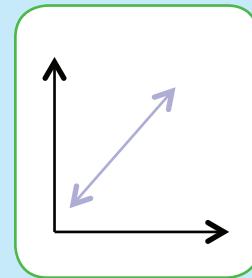
Increasing



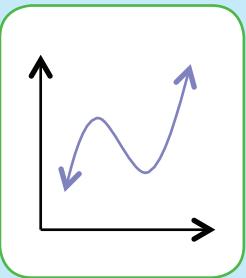
Decreasing



Constant



Linear



Non-linear

Can you get a non-linear increasing graph?

Can you get a non-linear decreasing graph?

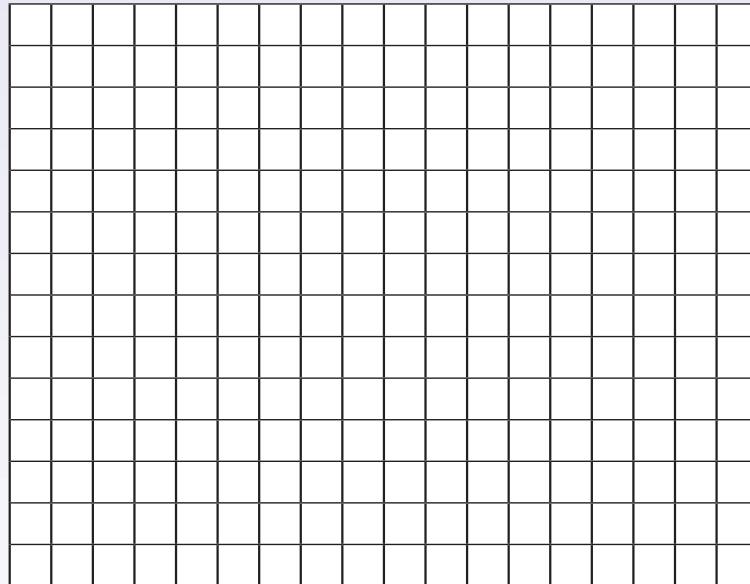
1. Draw graphs from the following tables. Describe each graph using the words **increasing, decreasing, constant, linear and non-linear**.

a. Thabo's brisk walking results.

The time walked was recorded after 2, 4, 6, 8 and 10 km.



Km	Minutes
2	20
4	40
6	60
8	80
10	100



b. Susan's brisk walking results.

The time walked was recorded after 2, 4, 6, 8 and 10 km.



Km	Minutes
2	20
4	45
6	50
8	75
10	95

c. Maximum and minimum average temperatures for my town for this year.

Month	Minimum in degrees Celsius	Maximum in degrees Celsius
January	27	14
February	25	14
March	24	12
April	22	10
May	19	9
June	17	8
July	16	7
August	17	8
September	22	9
October	23	12
November	25	13
December	28	14

A blank 10x10 grid for drawing or plotting. The grid consists of 100 small squares arranged in a single column and ten rows.

Be creative

Create your own table, draw a graph and describe it.

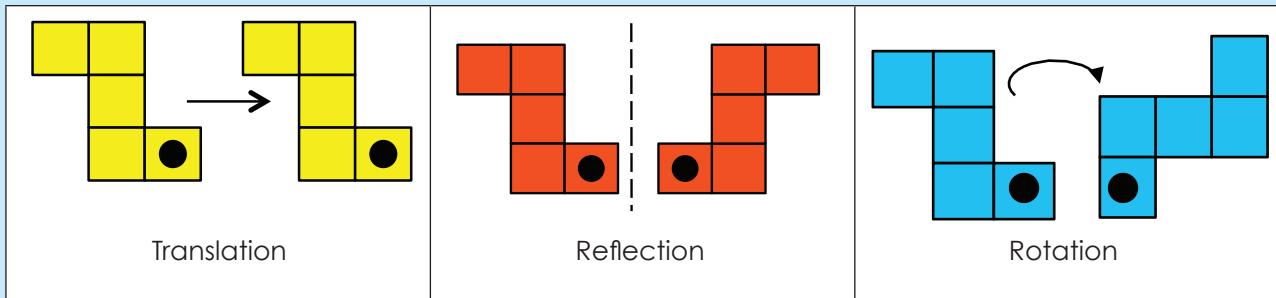




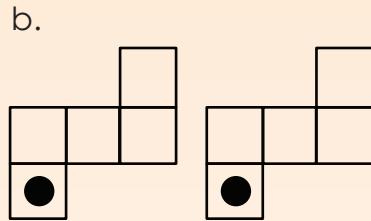
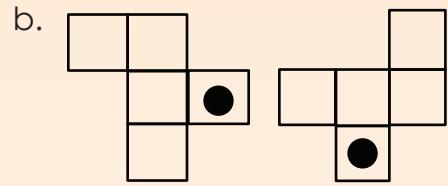
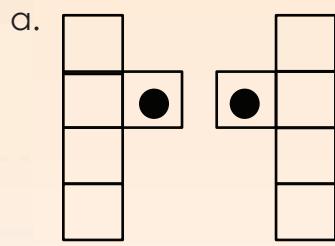
Transformations

Can you still remember?

Explain each transformation

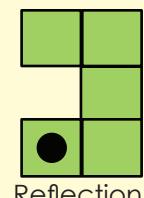
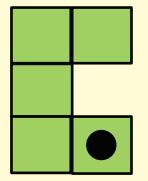
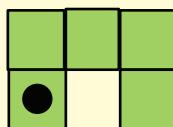
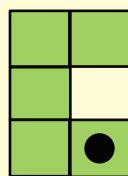


1. Tell how each figure was moved. Write translation, rotation, or reflection.



2. Label each shape as a translation, reflection or rotation.

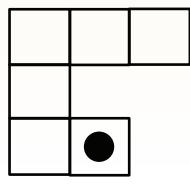
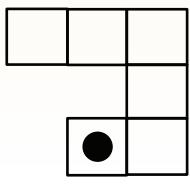
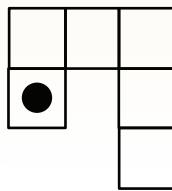
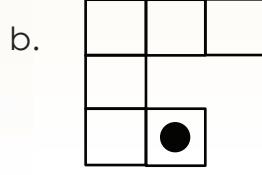
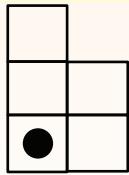
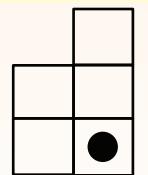
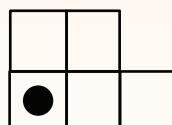
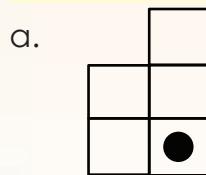
Example:



Rotation

Translation

Reflection



3. Create diagrams to show:

a. Rotation

A rotation is a transformation that moves points so that they stay the same distance from a fixed point (the the centre of rotation).

b. Reflection

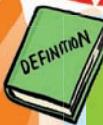
A reflection is a transformation that has the same effect as a mirror.

c. Translation

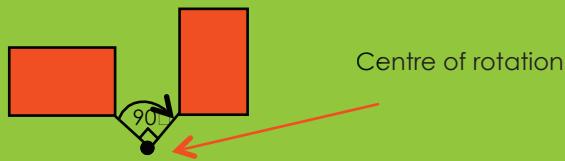
A translation is the movement of an object to a new position without changing its shape, size or orientation. When a shape is transformed by sliding it to a new position, without turning, it is said to have been translated.

Problem solving

Create a diagram using reflection, rotation and translation.



Rotation: a rotation is a transformation that moves points so that they stay the same distance from a fixed point, the centre of rotation.



Rotational symmetry: A figure has rotational symmetry if an outline of the turning figure matches its original shape.

Order of symmetry: This is how many times an outline matches the original in one full rotation.



Use any **recycled material** to demonstrate the difference between rotation and rotational symmetry.

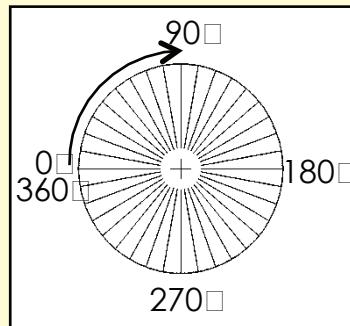
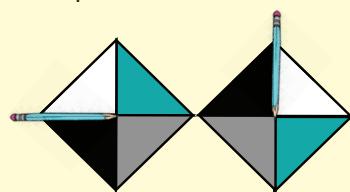
Rotation in nature and machines.



1. Look at the diagrams and explain them in your own words

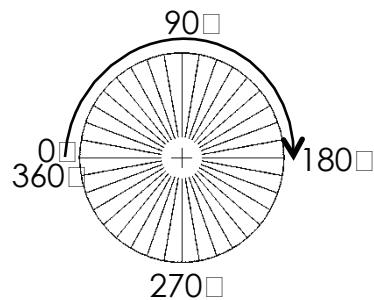
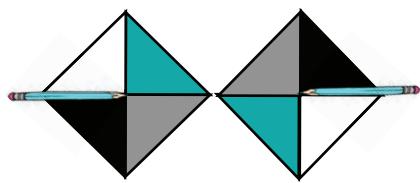
Example:

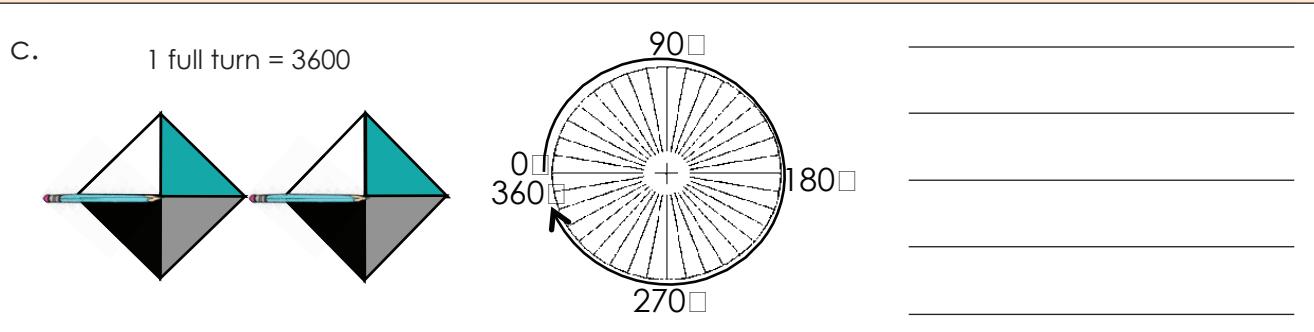
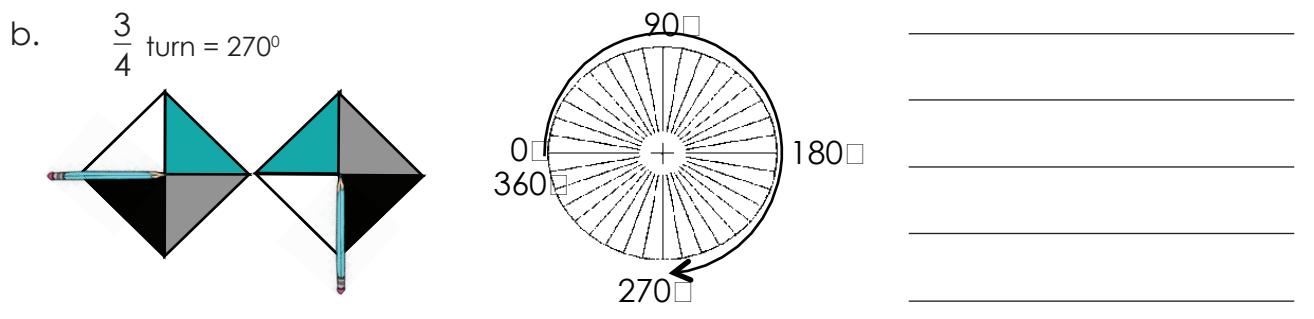
$$\frac{1}{4} \text{ turn} = 90^\circ$$



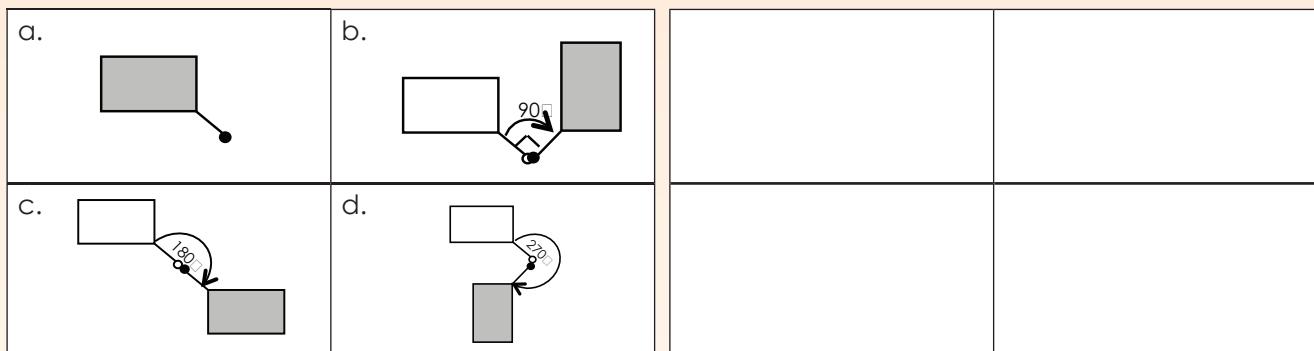
The paper rotated a quarter turn, which is the same as 90° . We can show it on a circular protractor.

a. $\frac{1}{2} \text{ turn} = 180^\circ$





2. Look at the drawings below and explain them.



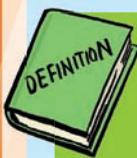
3. Complete the table below by rotating each shape.

	90°	180°	270°	360°

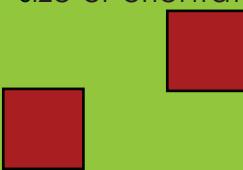
Problem solving

Make up your own rotations, with the centre of rotation outside the shape.

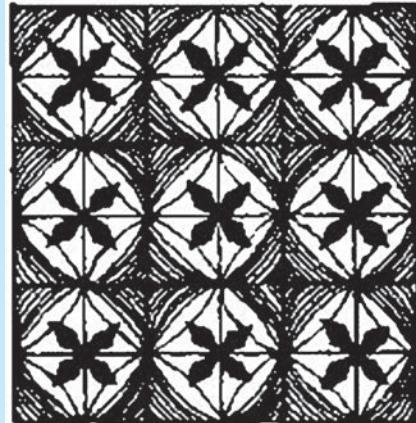




A translation is the movement of an object to a new position without changing its shape, size or orientation.

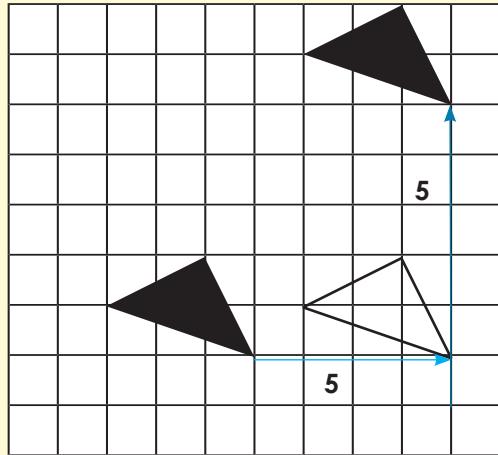


When a shape is transformed by sliding it to a new position, without turning, it is said to have been translated.

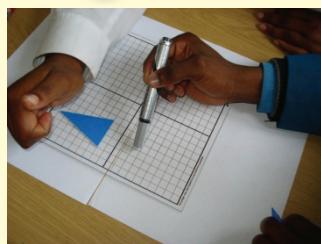


1. Explain each translation in your own words. The original shape is shaded.

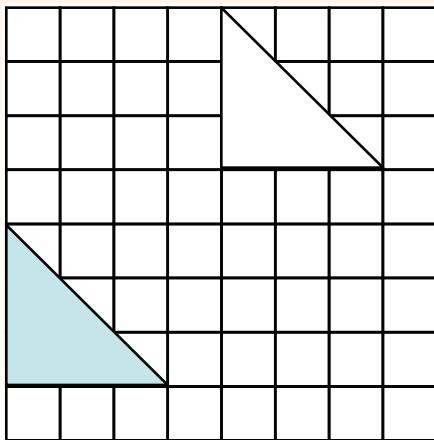
Example:



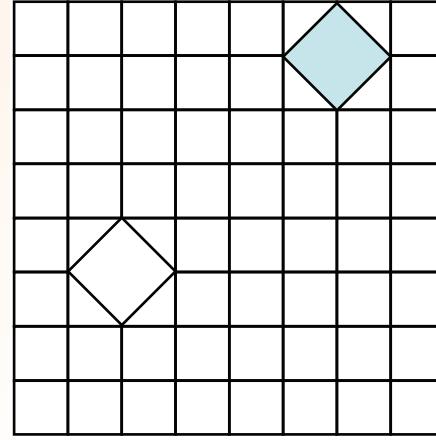
Each point of the triangle is translated four squares to the right and five squares up.



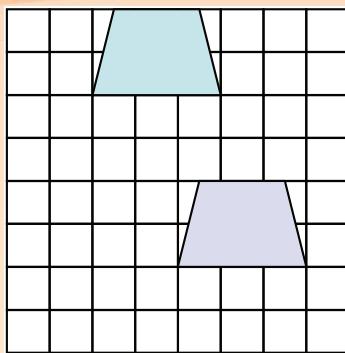
a.



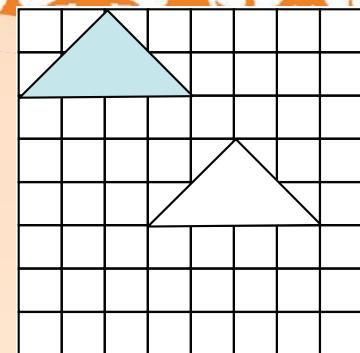
b.



c.

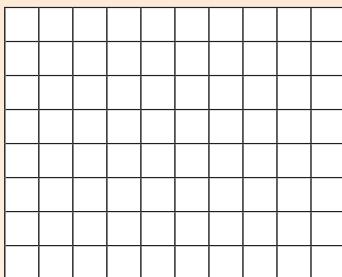


d.

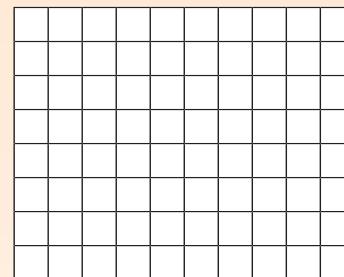


2. Show the following translations on a grid board.

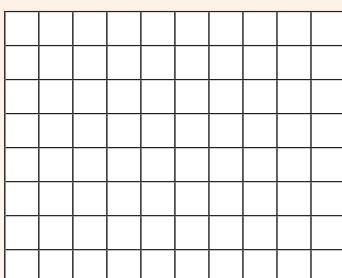
a. Each point of the triangle is translated four squares to the right and five squares up.



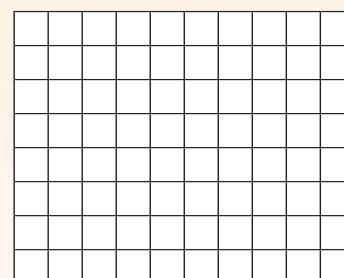
b. Each point of the rectangle is translated three squares to the left and three squares up.



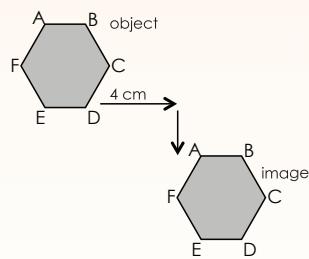
c. Each point of the triangle is translated five squares to the right and two squares down.



d. Each point of the square is translated two squares to the right and seven squares up.



3. In mathematics, the translation of an object is called its **image. Describe the translation below.**



Problem solving

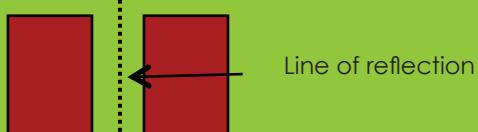
Find a translated pattern in nature and explain it in words.



Reflection and reflective symmetry

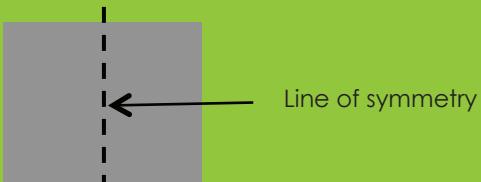


Reflection: a reflection is a transformation that has the same effect as a mirror.



Reflective symmetry

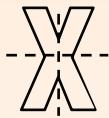
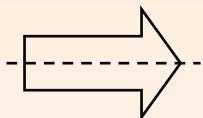
An object is symmetrical when one half is a mirror image of the other half.



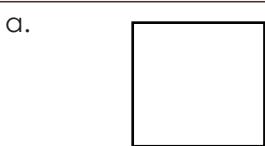
Look at the photograph. What do you see?



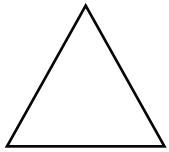
1. How many lines of symmetry does each have?



2. Draw all the lines of symmetry for each figure.

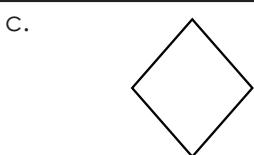


a.



b.

b.



c.

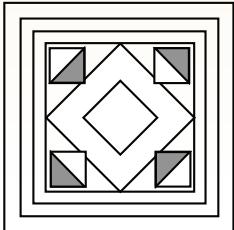


d.

b.

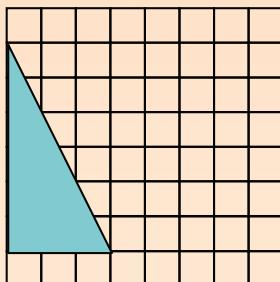


3. The following design uses reflective symmetry. One half is a reflection of the other half. The two halves are exactly alike and fit perfectly on top of each other when the design is folded correctly. How many lines of symmetry are there?

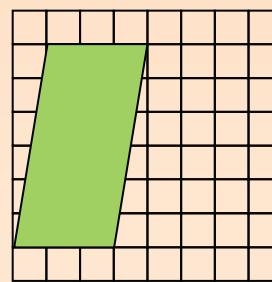


4. Show reflection using the geometric figure given. Remember to show the line of reflection.

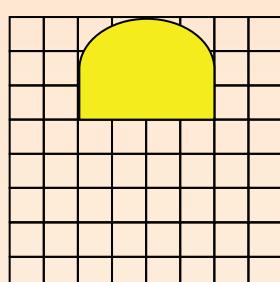
a.



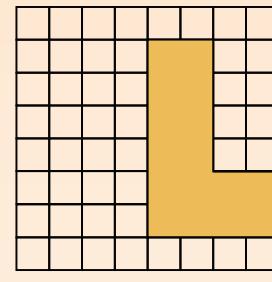
b.



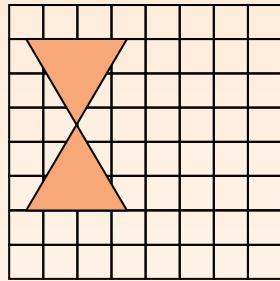
c.



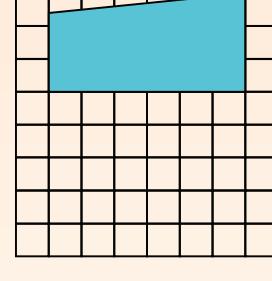
d.



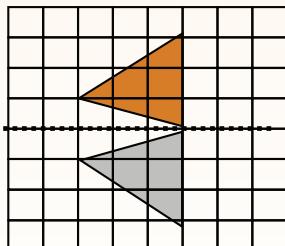
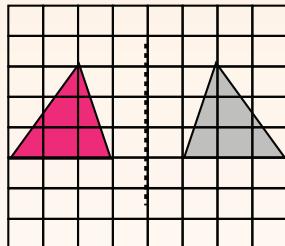
e.



f.



5. Look at the reflections and describe them.



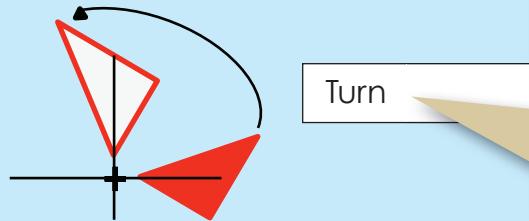
Problem solving

Find a photograph of reflection in nature.



Copy each transformation on grid paper and then explain it in words.

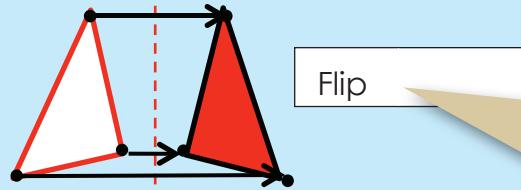
Rotation



Turn

Turning around a **centre**. The distance from the centre **to any point** on the shape stays the **same**. Every point makes a **circle** **around the centre** (rotation).

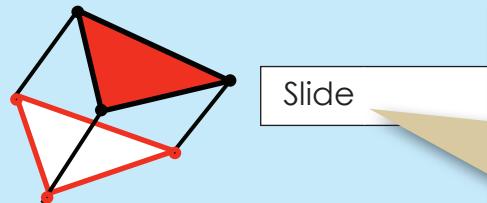
Reflection



Flip

It is a **flip** over a **line**. Every point is the same distance from the **centre line**. It has the same **size** as the **original image**. The shape stays the **same** (reflection).

Translation



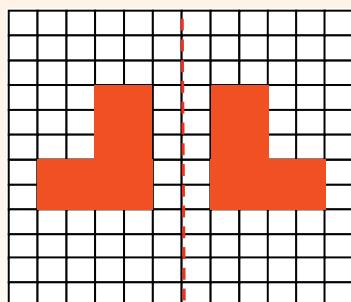
Slide

It means **moving** without rotating, flipping or resizing. Every point of the shape must move the **same distance** and in the **same direction** (translation).

1. Describe each diagram. Make use of words such as mirror, shape, original shape, line of reflection and vertical.

Reflection

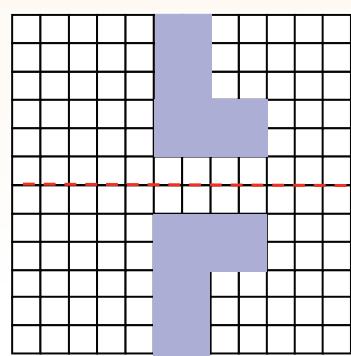
a.



When a shape is reflected in a mirror line, the reflection is the same distance from the line of reflection as the original shape.

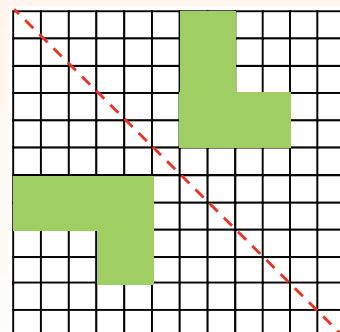
a.

b.



b.

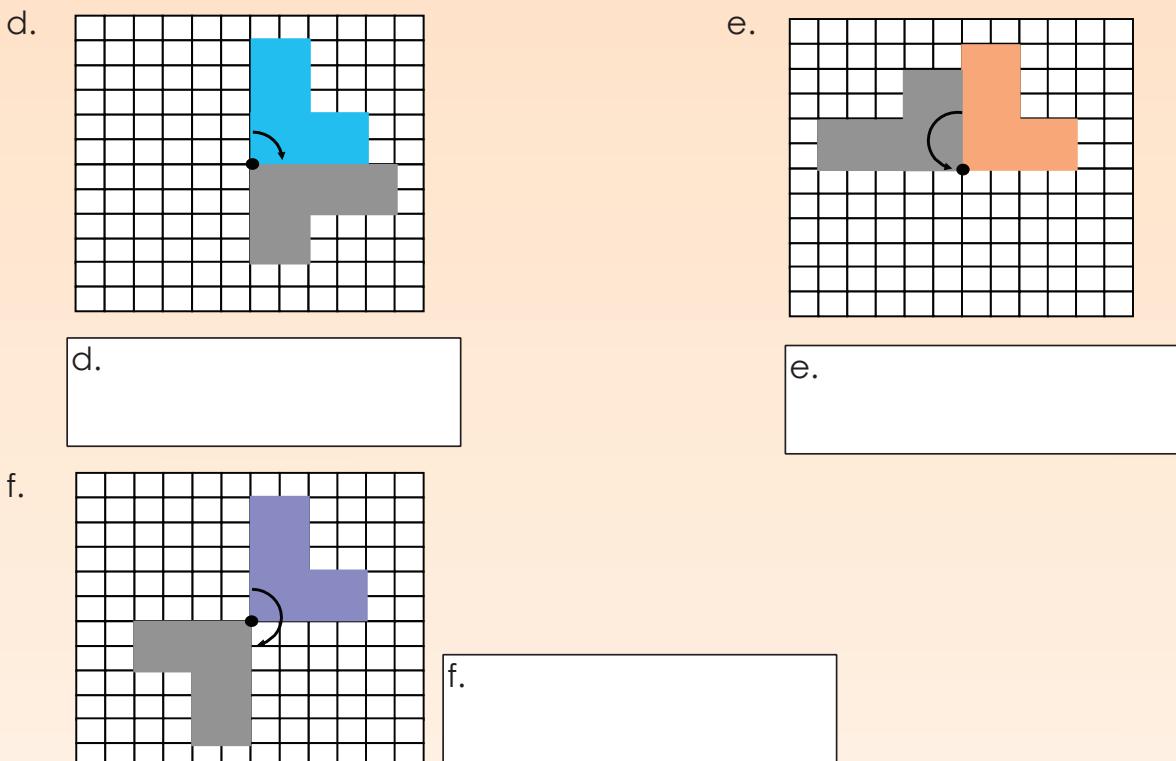
c.



c.

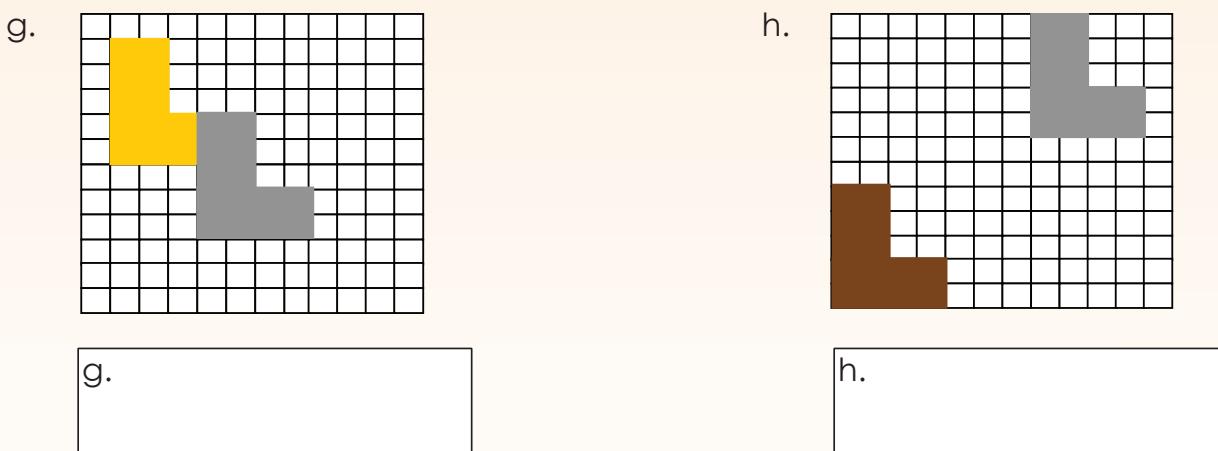
Rotation

Make use of words such as rotated or turned, clockwise, anti-clockwise, point of rotation and distance.



Translation

Make use of words such as shape, slide, one place to another, no turning, left, right, up, down, etc.



Share with your family

Draw the following on a grid and then describe the transformation:

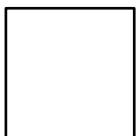
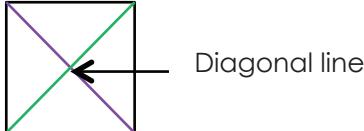
- reflection
- rotation
- translation

When we do an investigation we should:

- spend enough time exploring problems in depth
- find more than one solution to many problems
- develop your own strategies and approaches, based on your knowledge and understanding of mathematical relationships
- choose from a variety of concrete materials and appropriate resources
- express your mathematical thinking through drawing, writing and talking

1. Prove that the diagonal of a square is not equal to the length of any of its sides.

a. What do I know? Make a drawing of each.

What transformation is (rotation, reflection, and translation).	What a square is. 
What diagonal lines of a square are. 	That all the sides of a square are equal in length.

b. What do I want?

To compare the length of a side of a square with the length of a diagonal.
I can/must use rotation, translation and/or reflection.

c. What do I need to introduce? Make a drawing of each.

Note that sometimes we think of something later on; we don't always think of everything at the beginning. Therefore people will have different answers here.

A line of reflection.	A point of rotation.	A grid on which to measure translation.
-----------------------	----------------------	-----------------------------------------

d. Attack

You often get “stuck” and are tempted to give up. However, this is the exact point at which it is critical for you to use the time and space to get through the point of frustration and look for alternative ideas. This is the phase in which you make conjectures, collect data, discover patterns and try to convince or justify your answers.

Remember to use the information in a, b and c.

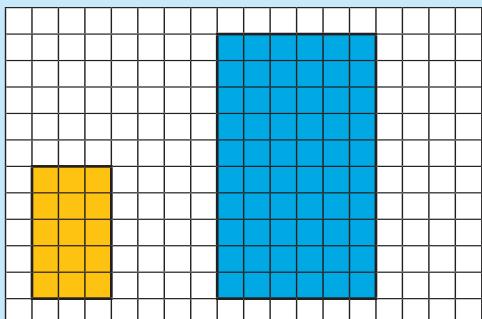
e. Review

Check your conclusions or resolutions, reflect on what you did – the key ideas and key moments.

Family time

Share this investigation with a family member.

Look at this diagram and discuss.



Orange rectangle

The length = 5 The width = 3

Blue rectangle

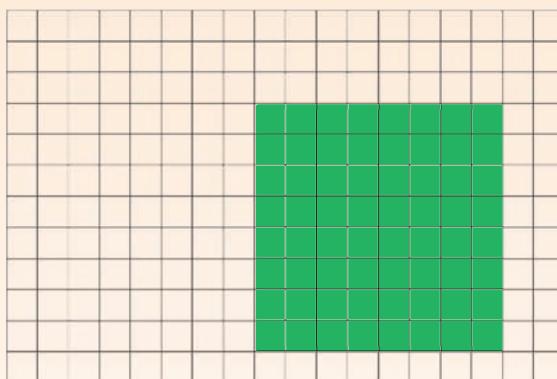
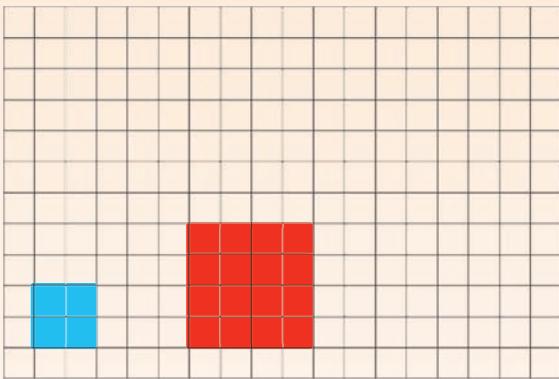
The length = 10 The width = 6

The length of the **blue rectangle** is two times/twice the length of the **orange rectangle**.

The width of the **blue rectangle** is two times/twice the width of the **orange rectangle**.

The **orange rectangle** is two times/twice enlarged.

1. Use the diagrams to answer the questions.



Blue square	Red square	Green square
Length = <u>2</u>	Length = <u>4</u>	Length = <u>8</u>
Width = <u>2</u>	Width = <u>4</u>	Width = <u>8</u>

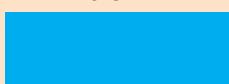
b. The length of the red square is 2 times the length of the blue square.
 The width of the red square is 2 times the width of the blue square.
 The red square is enlarged 2 times.

c. The length of the green square is 4 times the length of the red square rectangle.
 The width of the green square is 4 times the width of the red square.
 The green square is enlarged 4 times.

d. The length of the green square is 8 times the length of the blue square.
 The width of the green square is 8 times the width of the blue square.
 The blue square is reduced 8 times.

2. Use the diagrams to answer the questions.

3 cm



Blue rectangle:

The length = ____
The width = ____

6 cm



Red rectangle:

The length = ____
The width = ____

24 cm



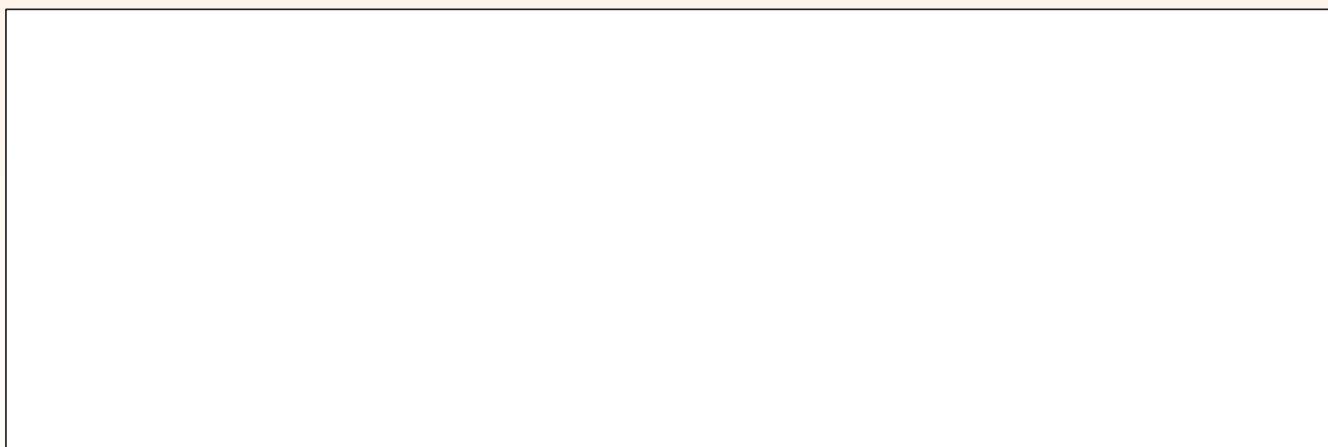
Green rectangle:

The length = ____
The width = ____

Compared to the:

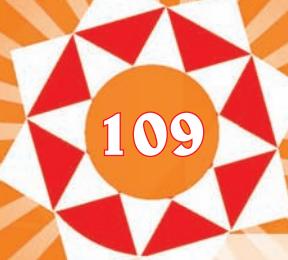
- Red rectangle, the blue rectangle is reduced ____ times.
- Green rectangle, the blue rectangle is reduced ____ times.
- Blue rectangle, the red rectangle is enlarged ____ times.
- Green rectangle, the red rectangle is enlarged ____ times.
- Blue rectangle, the green rectangle is enlarged ____ times.
- Red rectangle, the green rectangle is enlarged ____ times.

3. Draw a 1 cm by 2 cm rectangle. Enlarge it twice and then enlarge the second rectangle six times. Make a drawing to show your answer.



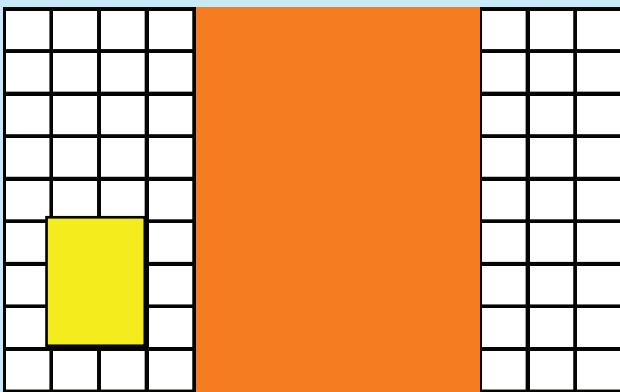
Problem solving

What will the perimeter of a 20 mm by 40 mm rectangle be if you enlarge it by 3?



Drawing graphs

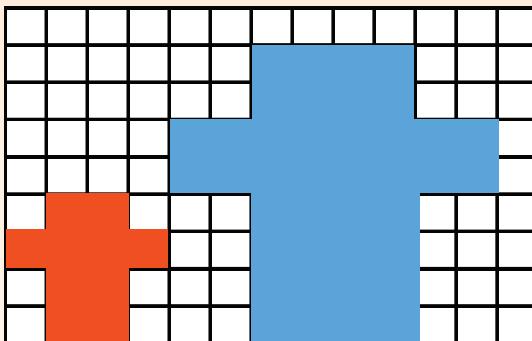
How do you know this figure is enlarged by 3?



We say the scale factor is 3.

The scale factor from small to large is 3.
The scale factor from large to small is 3.

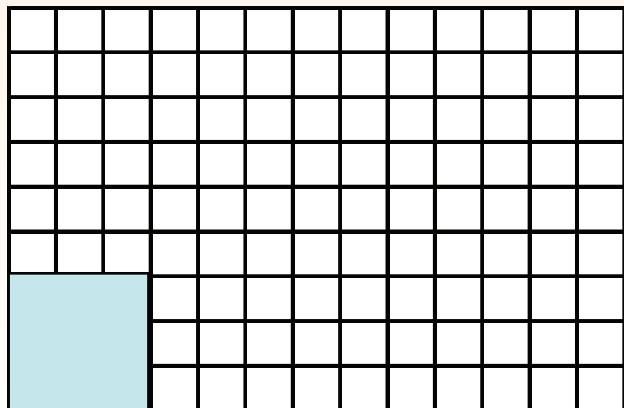
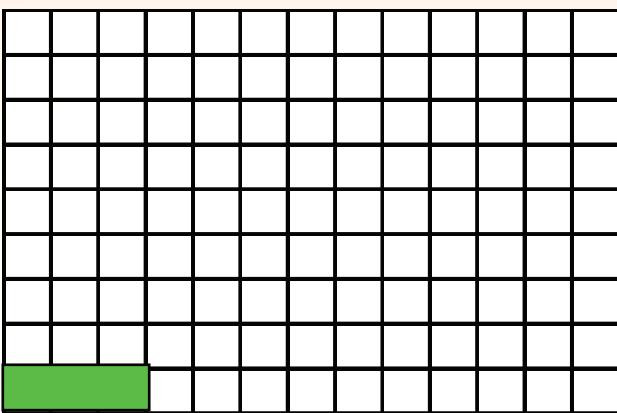
1. By what is this shape enlarged? Write down all the steps.



2. Enlarge the rectangle by:

a. scale factor 4

b.



3. Complete the table. Start with the original geometric figure every time.

Geometric figure	Enlarge by scale factor 2.	Enlarge by scale factor 5.	Enlarge by scale factor 10.
a. $2 \text{ cm} \times 3 \text{ cm}$  $= 6 \text{ cm}^2$	$2 \text{ cm} \times 2 \times 3 \text{ cm} \times 2$  $= 24 \text{ cm}^2$	$2 \text{ cm} \times 5 \times 3 \text{ cm} \times 5$  $= 150 \text{ cm}^2$	$2 \text{ cm} \times 10 \times 3 \text{ cm} \times 10$  $= 6 \text{ cm}^2$
b. $5 \text{ cm} \times 1 \text{ cm}$			
c. $4 \text{ cm} \times 2 \text{ cm}$			
d. $8 \text{ cm} \times 3 \text{ cm}$			
e. $1,5 \text{ cm} \times 2 \text{ cm}$			

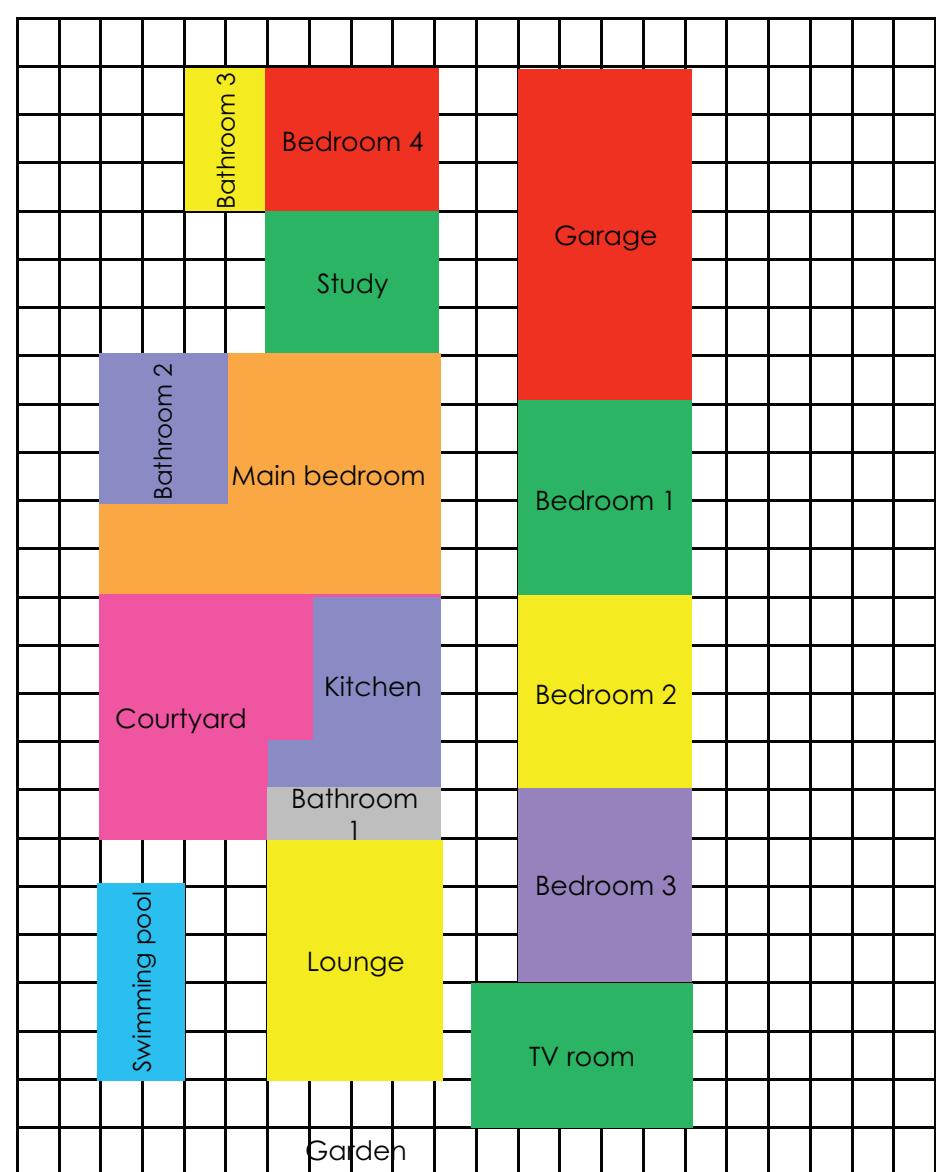
Problem solving

Enlarge a $1,5 \text{ cm} \times 5 \text{ cm}$ geometric figure by scale factor 3.

Use your knowledge gained in the previous two lessons. You might need to revise the following words:

- enlargement
- reduction
- scale factor

A client asked you to make the following amendments to the house plan.



1. Enlarge the following by scale factor 2.

- Garage
- Bedroom 3

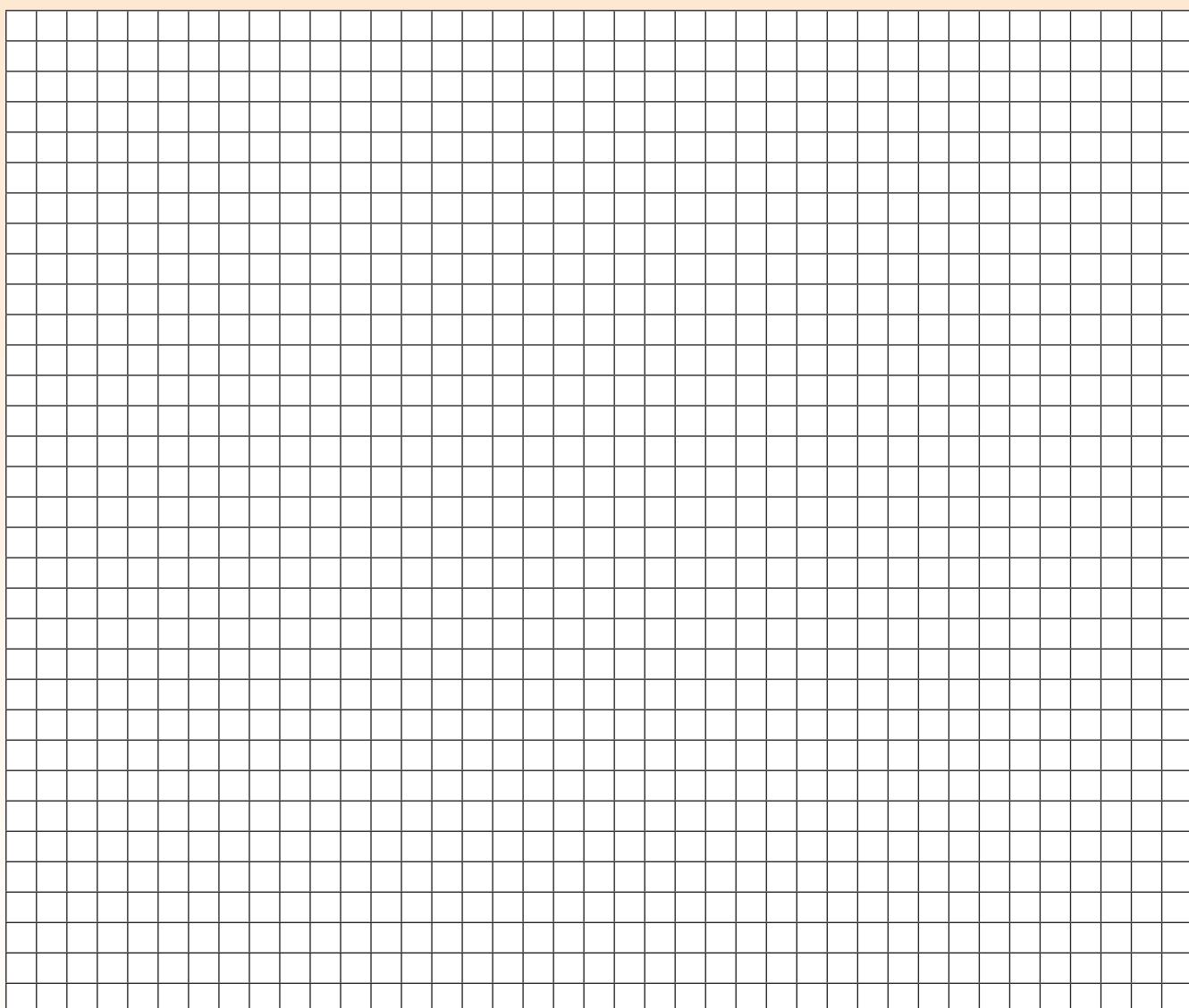
2. Reduce the following by scale factor 2.

- a. Bedroom 1
- b. Bedroom 2

3. Enlarge by scale factor 3.

- a. TV room
- b. Study (remove bedroom 4)

4. The client wants to build a Lapa that is reduced by the swimming pool's scale factor 2.



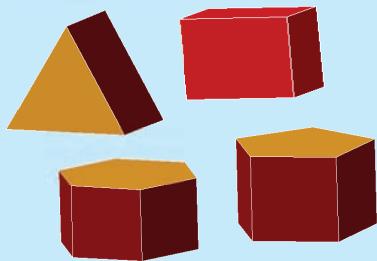
Problem solving

Design your dream house. Enlarge it by scale factor 2.

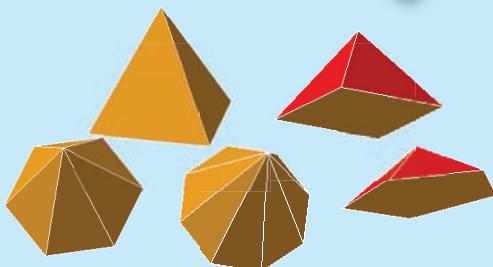


Prisms and pyramids

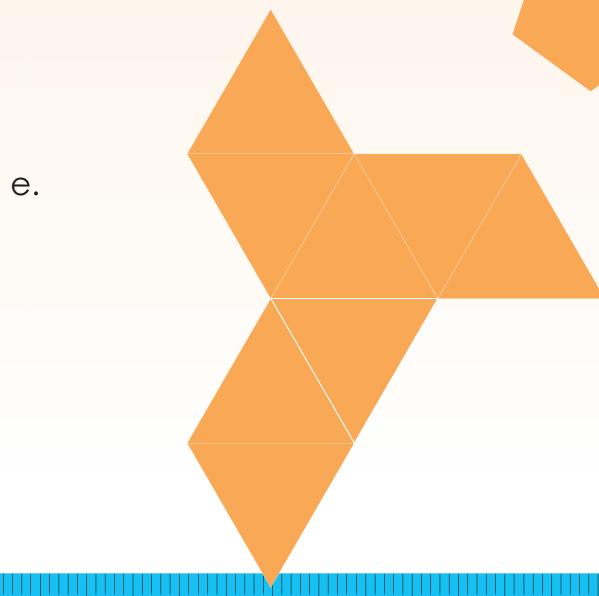
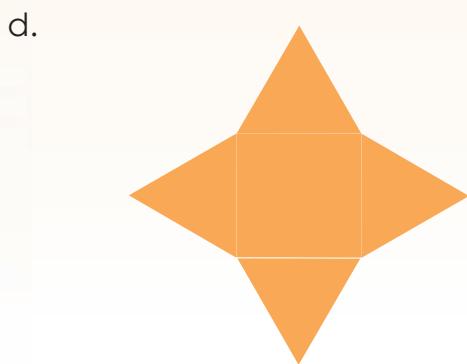
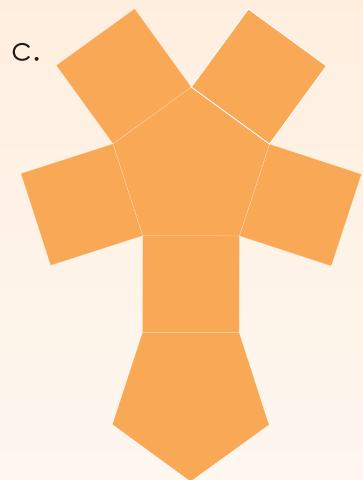
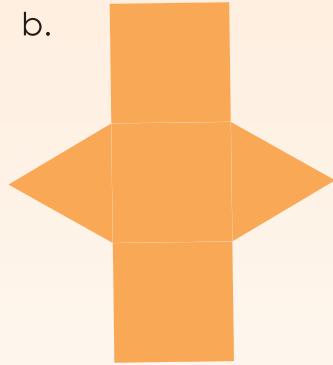
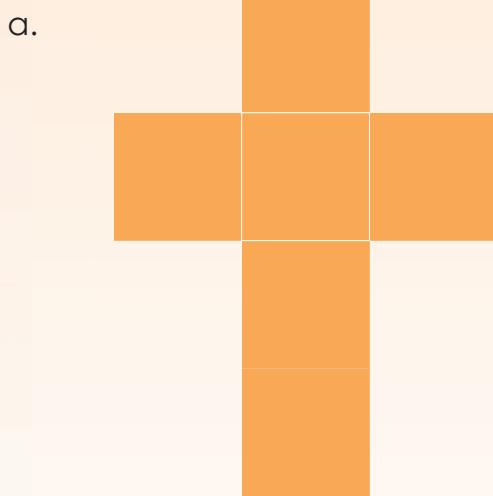
Why kinds of prisms are these?



Why kind of pyramids are these?



1. Make the following geometric objects using the nets below. Enlarge the nets by a scale factor of 2. You will need some grid paper, ruler, sticky tape and a pair of scissors.



2. Identify and name all the geometric solids (3-D objects) in these diagrams.

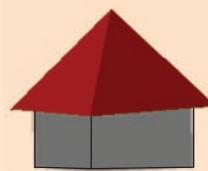
a.



b.

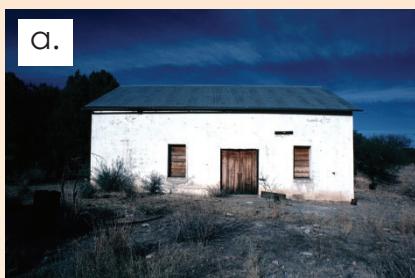


c.



3. Identify, name and label as many pyramids and prisms as you can.

a.



b.



c.



4. Compare prisms and pyramids.

Prisms	Pyramids



Problem solving

Name five pairs of a pyramid and a prism that will exactly fit on top of each other, and say why.

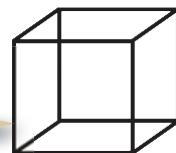


3-D objects

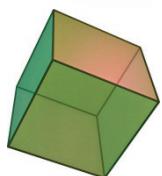
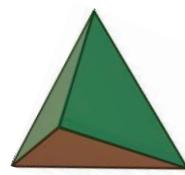
This is a skeleton of a tetrahedron.



This is a skeleton of a cube.

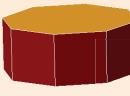


A tetrahedron is a special type of triangular pyramid made up of identical triangles.



A hexahedron (plural: hexahedra) is a polyhedron with six faces. A regular hexahedron, with all its faces square, is a cube.

1. Which pyramid will fit exactly onto each prism? Draw lines to show it.



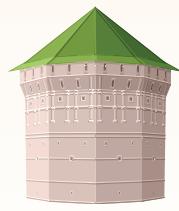
a. Circle the tetrahedron in blue.
b. Circle the hexahedron in red.

2. Describe the prisms and pyramids in these pictures.

a.



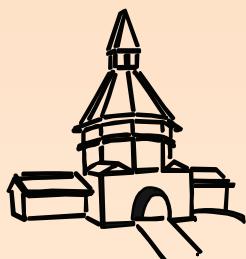
b.



c.



3. Your friend made this drawing of a building she saw. Identify and name the solids.



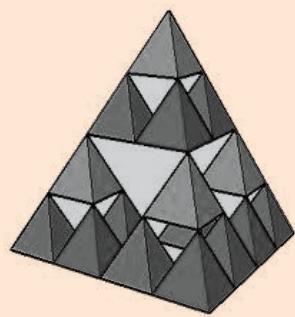
4. Draw the nets for the following:

Tetrahedron

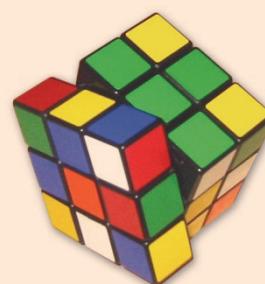
Hexahedron

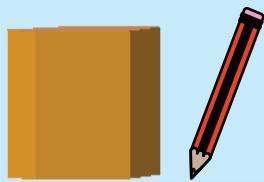
Problem solving

How many tetrahedrons do you need to complete the big tetrahedron?



How would you use the word hexahedron to describe this Rubic cube?

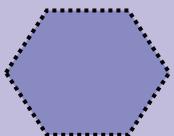


Geometric solid

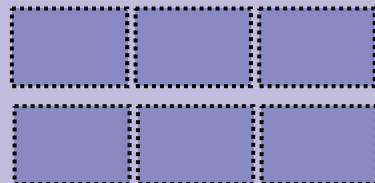
This is what we get if we trace around each face of the hexagonal prism.

Geometric figures

2 hexagons



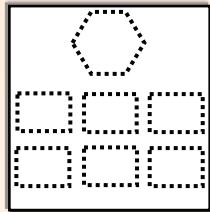
6 rectangles



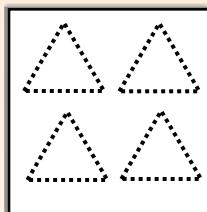
A 2-D shape is a "geometric figure" and a 3-D object is a "geometric solid".

1. Which geometric solid can be made with these geometric figures?

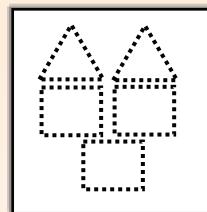
a.



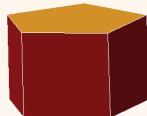
b.



c.

**2. Identify all the geometric figures in these solids and make a drawing of all the shapes.**

a.



b.

**3. a. Make various geometric solids using geometric figures using waste products.**

- prisms (triangular prism, cube, rectangular, pentagonal, hexagonal and octagonal)
- pyramids (triangular, tetrahedron, rectangular, pentagonal, hexagonal and octagonal)

b. Use the geometric solids to create "buildings of the future".

4. a. Write down how you created each polyhedron, focusing on the shapes of the faces and how you joined them. You may include drawings.
- b. Write a description of how you have put the geometric solids together to create your “buildings of the future”. Give reasons why you have used certain solids for certain buildings.
- c. Present your work to the class.

Presentation Tips

When presenting you should:

- Make eye contact with different people throughout the presentation;
- Start by explaining what the content of presentation is about;
- Use natural hand gestures to demonstrate;
- Stand up straight with both feet firmly on the ground;
- Demonstrate a strong positive feeling about the topic during the entire presentation;
- Stay within the required time frame;
- Use visual aids to enhance the presentation;
- Explain all points thoroughly;
- Organise your presentation well and maintain the interest level of the audience.

Problem solving

Fit two geometric solids on top of each other. Where they touch the faces should be the same. The two geometric solids cannot be prisms or pyramids.

What geometric solid is it?



All the faces are flat.

I count five faces.

Two are triangles and three are rectangles.

Do the following in pairs. Alternate the questions amongst yourselves.

1. Ask your friend to close his or her eyes. Then ask him or her the following questions:

a. Name and describe the **new solid**.

Imagine you have a cube.

Imagine you now have two identical cubes.

Place them together.

After imagining the object, draw, name and describe it.

Draw:

Draw:

Describe:

b. Name and describe the solid from **different views**.

Imagine you are looking at a large cardboard box that looks like a cube.

Can you stand so that you can see only one square?

Can you stand so that you can see 2 or 3 squares?

Seeing one square

Seeing two squares

Seeing three squares

The pyramids are the stone tombs of Egypt's kings - the Pharaohs. They have stood for thousands of years, filled with many clues about what life (and death) was like in Ancient Egypt.

What is the great pyramid of Giza? Find out?

Great pyramid of Giza and maths.

- The base originally measured about 230,33 m square.
- The original height was 146,59 m.
- A total of over 2 300 000 stone blocks of limestone and granite were used.
- The construction date was about 2589 B.C.
- Estimated construction time was 20 years.
- Estimated total weight is 6,5 million tons.

c. What type of pyramid (geometric objects) will we mostly find in Egypt?

d. Name and describe the solid from **different views**.

Imagine you are visiting the pyramids in Egypt.

You are standing on the ground, looking at a pyramid.

What is the maximum number of triangles you see?

What if you were in an aeroplane flying overhead?

e. Name and describe the solid from **different views**.

View from the ground

Aerial view

An aerial view is also called a bird's eye view. Why do you think it is named this?



Problem solving

Describe a geometric solid to your family and ask them to imagine it.

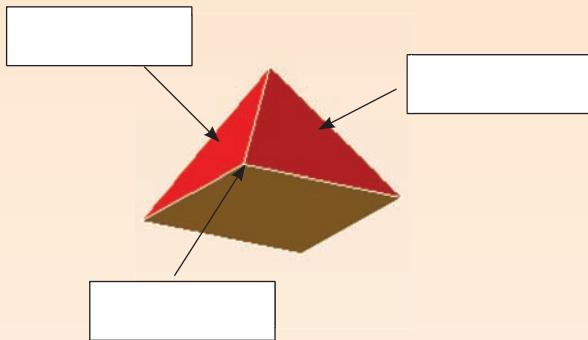
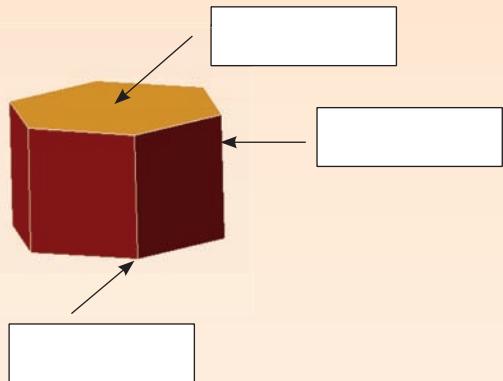


Surface: A surface may be flat or curved. We can also call it a face.

Vertex (plural: vertices): A point where three surfaces meet (corner).

Edge: Where two surfaces are joined.

1. Label the following using the words: **surface (face)**, **edge** and **vertex**.



2. Label the surfaces, vertices and edges on each photograph.

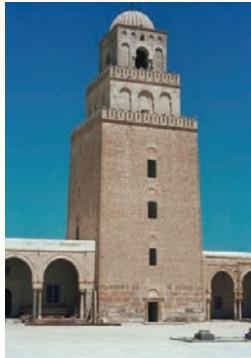
a.



b.



c.



d. Mark the apex on each building with a star(*) .

An apex is the highest point of a geometric solid with respect to a line or plane chosen as base.

3. What do all these objects have in common? When closed, they all have:



- ___ faces
- ___ edges
- ___ vertices



- ___ faces
- ___ edges
- ___ vertices



- ___ faces
- ___ edges
- ___ vertices



- ___ faces
- ___ edges
- ___ vertices

4. Label the following using the words: surface (face), edge and vertex. Also say which geometric object each one will form.



Geometric object:

- ___ edges
- ___ vertices
- ___ faces



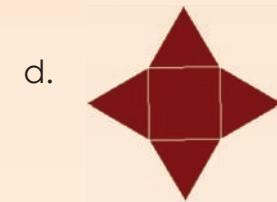
Geometric object:

- ___ edges
- ___ vertices
- ___ faces



Geometric object:

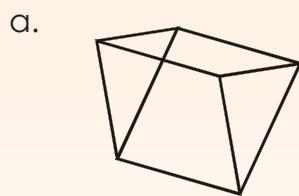
- ___ edges
- ___ vertices
- ___ faces



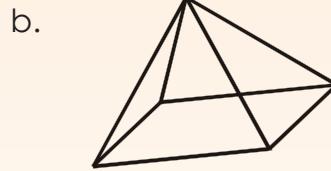
Geometric object:

- ___ edges
- ___ vertices
- ___ faces

5. Look at these skeletons. Say how many vertices and edges you see in each structure



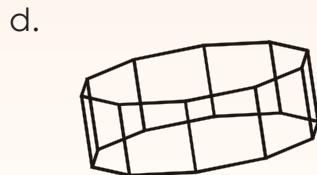
___ vertices ___ edges



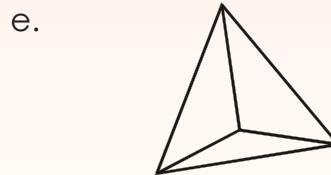
___ vertices ___ edges



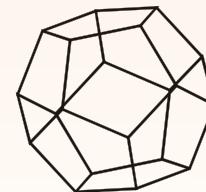
___ vertices ___ edges



___ vertices ___ edges



___ vertices ___ edges

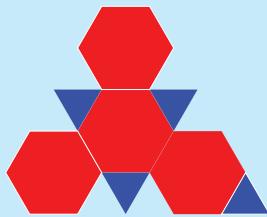
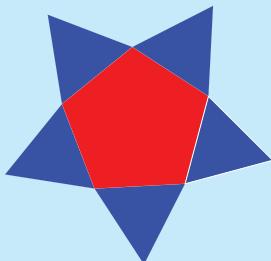
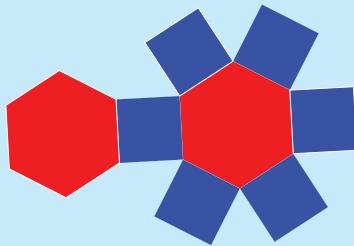


___ vertices ___ edges

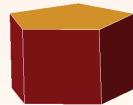
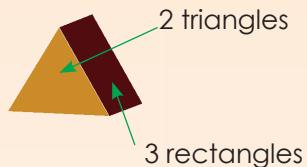
Problem solving

- Can a prism have an odd number of vertices? Give an example.
- Can a pyramid have an odd number of vertices?
- How many more faces does an octagonal pyramid have than a heptagonal pyramid?

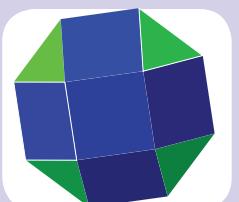
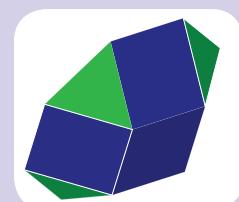
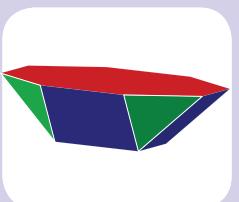
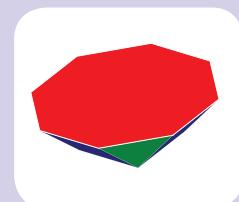
Think!!! Look at these **nets** of geometric solids. How many surfaces, vertices and edges does each solid have?



1. Write labels with arrows pointing to the geometrical figures which you can see in each object, and write how many of each there are.



Identify all the geometric figures in this geometric solid. We provide you with four views of the geometric solid to help you.



2.

	Name of solid	Shapes made of	No. of edges	No. of vertices	No. of surfaces
					
					
					
					
					
					

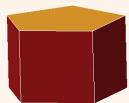
a. Look at the table above and compare a triangular pyramid and a square pyramid. Describe the *similarities* and differences between them.

b. Describe the differences between a hexagonal prism and an octagonal prism.

c. Describe the differences between a hexagonal pyramid and an octagonal pyramid.

d. What should you do to the geometric solid on the left to change it to the geometric solid on the right?

i.





ii.





Solve this with a family member.

Describe the geometric solid using the words surfaces (faces), vertices and edges.

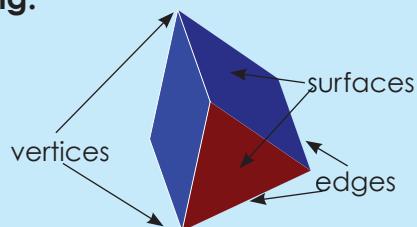
We give you the unfoldings to help you to solve this.



Even more surfaces, edges and vertices

Revise the following:

- surfaces (faces)
- vertices
- edges



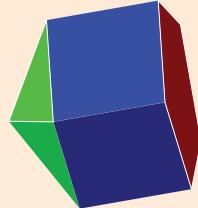
Identify the surfaces, vertices and edges in this photograph.



1. Look at the different polyhedra.

Identify the surfaces (faces), vertices and edges.

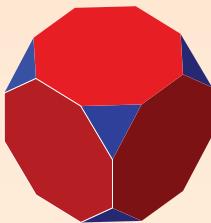
a.



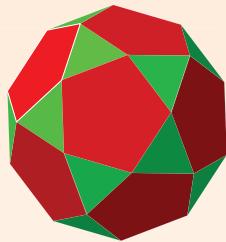
b.



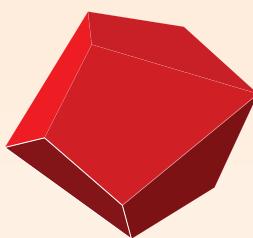
c.



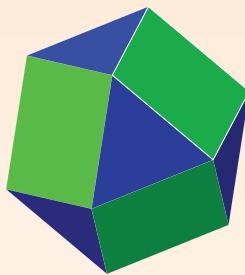
d.



e.



f.



2. Visualise how many vertices a pentagonal prism has. _____

a. How many edges does it have? _____

b. How many faces? _____

c. What about a heptagonal prism? _____

d. Heptagonal pyramid? _____

3. Complete the table

	Solid	Vertices	Edges	Faces	Calculate $F - E + V$ for each geometric solid. F = faces, E = edges and V = vertices. What do you notice?
Triangular prism					
Rectangular prism					
Pentagonal prism					

	Solid	Vertices	Edges	Faces	Calculate $F - E + V$ for each geometric solid. F = faces, E = edges and V = vertices. What do you notice?
Hexagonal prism					
Octagonal prism					
Triangular prism					
Square pyramid					
Pentagonal pyramid					
Hexagonal pyramid					
Octagonal pyramid					

Now Euler's formula tells us that $V = E + F - 2$

or, in words: the number of vertices, minus the number of edges, plus the number of faces, is equal to two.

Euler's formula is true for most polyhedra. The only polyhedra for which it doesn't work are those that have holes running through them.

Classification

If mathematicians find a property that is true for a whole class of objects, they know that they have found something useful. They use this knowledge to investigate what properties an individual object can have and to identify properties that all of them must have.

- Is it possible to get a polyhedron with seven edges? _____
- Do you see any solid with 7 edges? _____
- Which solid has 6 edges? _____
- Note that every polyhedron has more than three faces. So try it with the formula:

$$F - E + V = 2$$

Example 1: $4 - 7 + 5 = 2$

Example 3: $6 - 7 + 3 = 2$

Example 5: $8 - 7 + 1 = 2$

Example 2: $5 - 7 + 4 = 2$

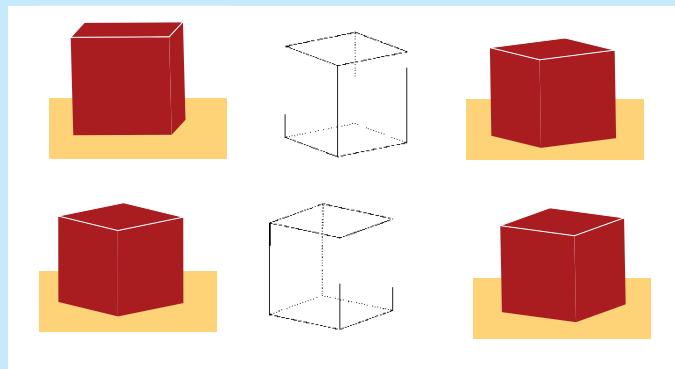
Example 4: $7 - 7 + 2 = 2$

Example 6: $9 - 7 + 0 = 2$

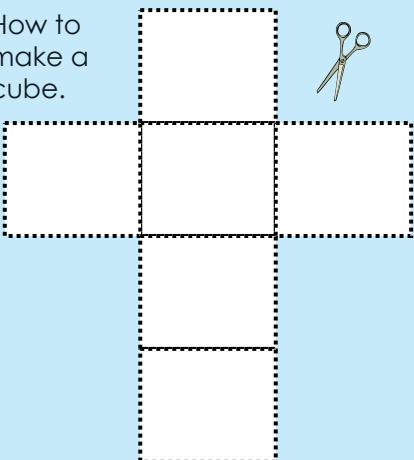
Problem solving

Investigation: Use Euler's formula with the problem solving solid in the previous worksheet.

In this activity you are going to look at the cube from different perspectives. Make a cube and put it in the same position in front of you.



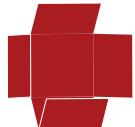
How to make a cube.



Step 1



Step 2



Step 3



Step 4



1. Draw and name each angle.

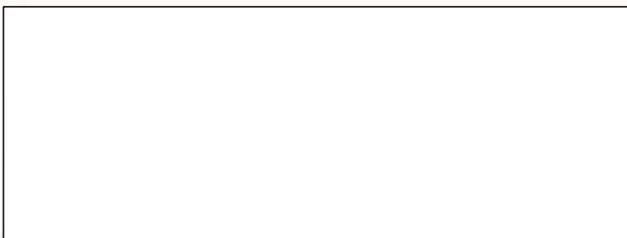
a. 40°



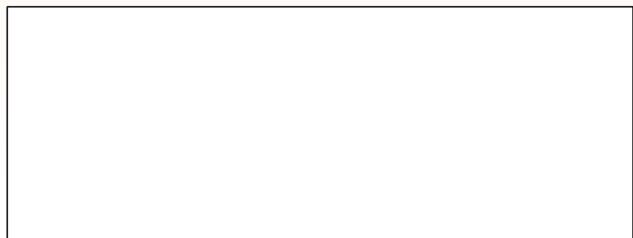
b. 90°



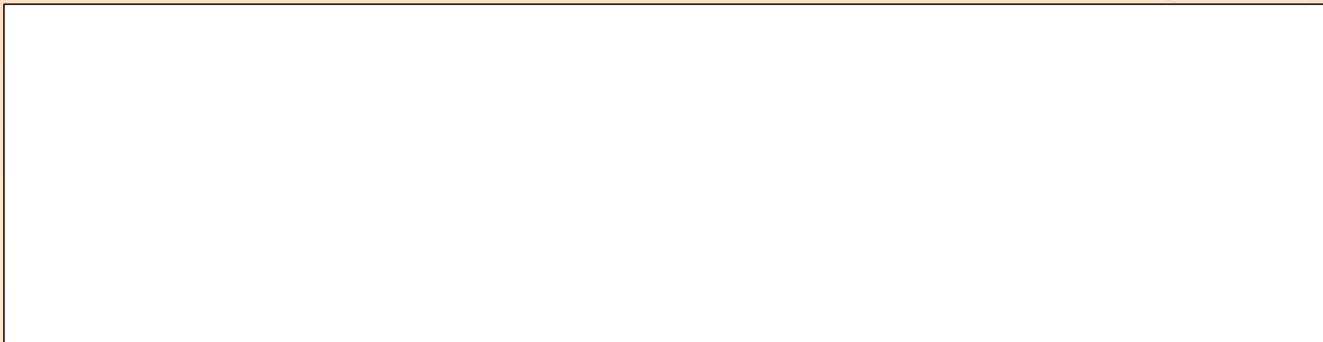
c. 120°



d. 150°

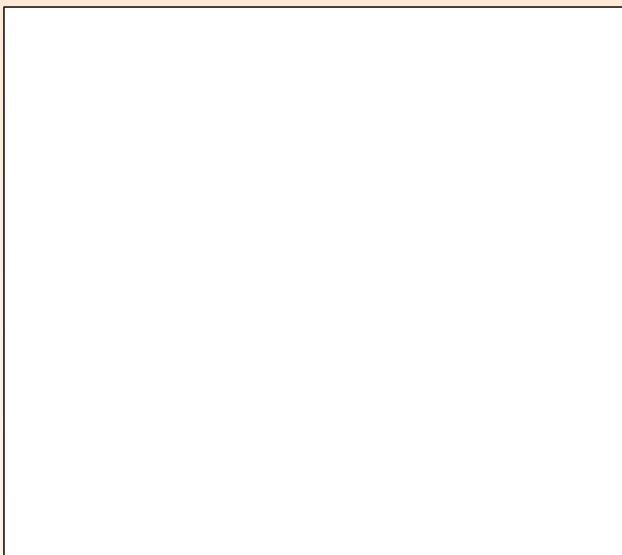


2. How will you draw an angle bigger than 180° ?

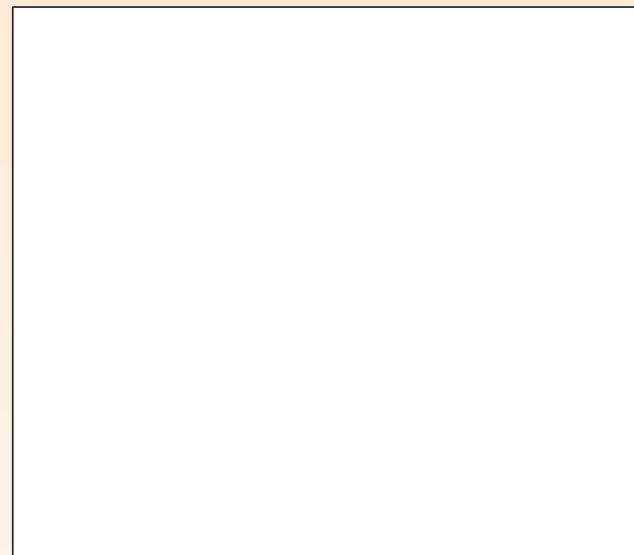


3. Draw and name each angle.

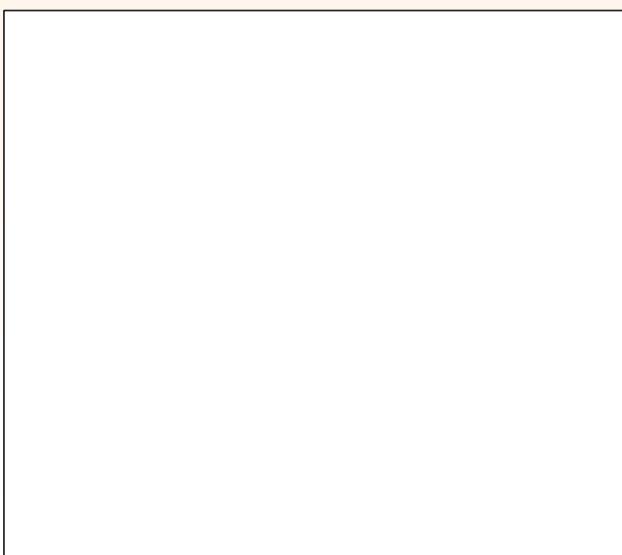
a. 190°



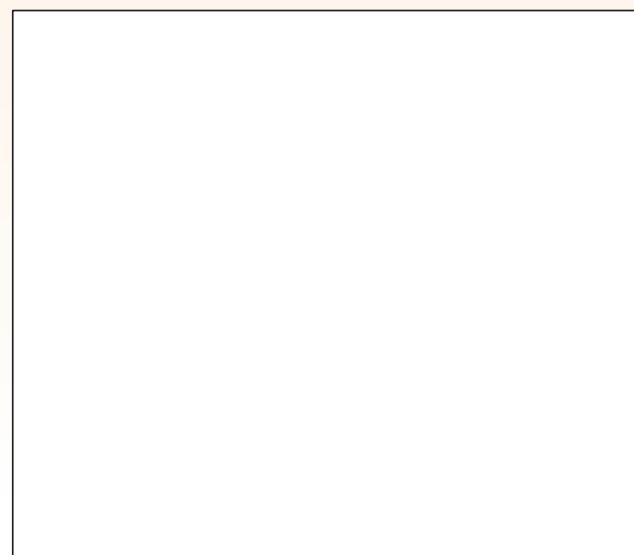
b. 280°



c. 300°



d. 275°

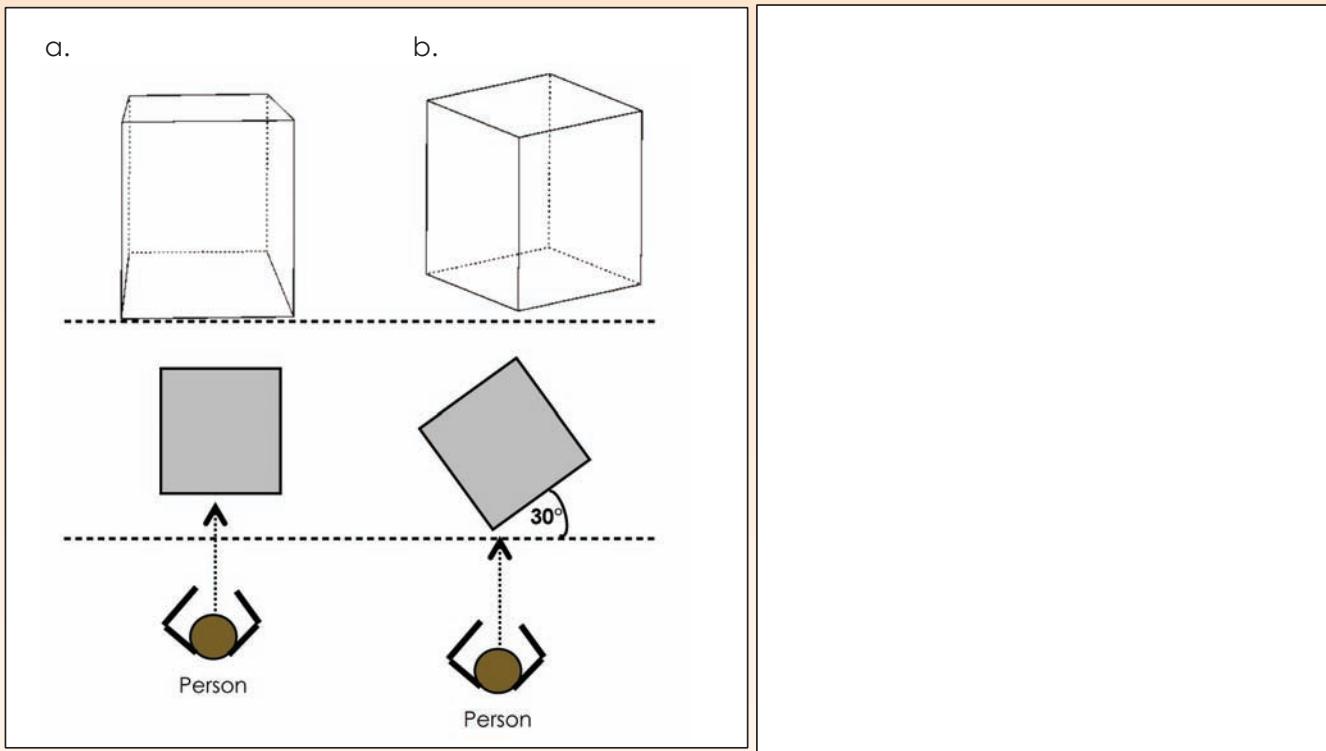


continued

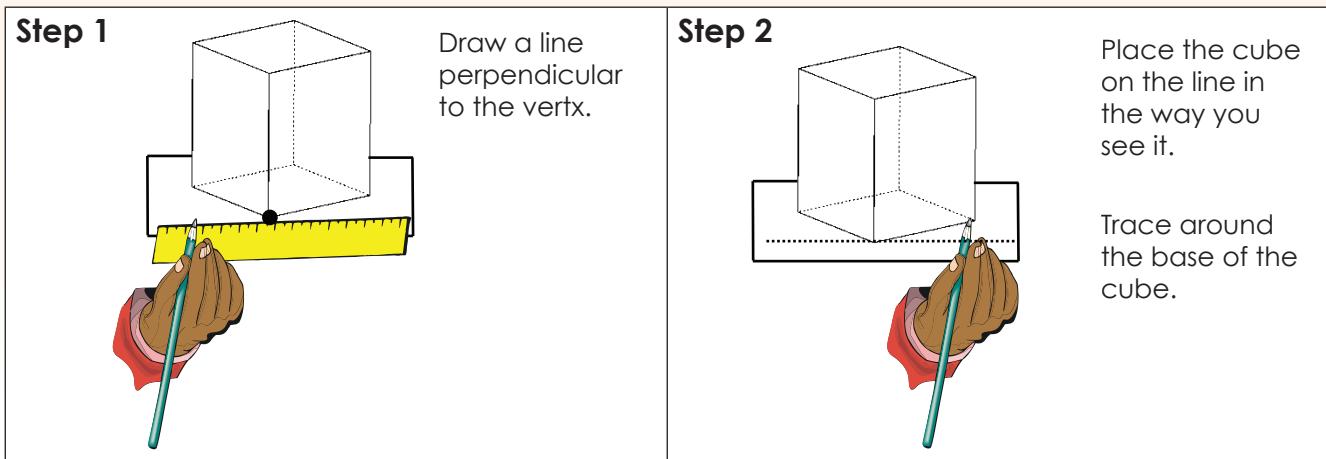
4. Look at the drawings below. Show and explain them.

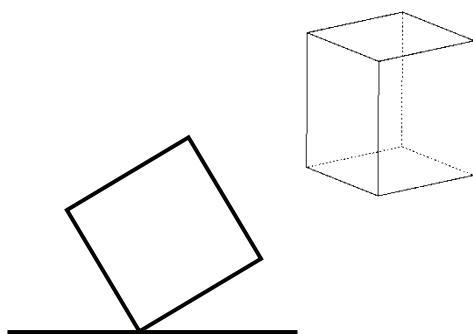
Do b. practically.

See if you can draw a cube with an angle of 30° as above, without a protractor. Place a cube on your desk and put a piece of paper under the cube.

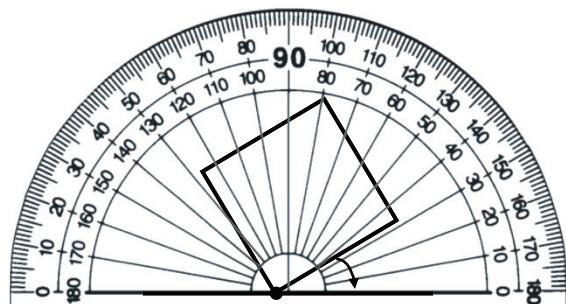
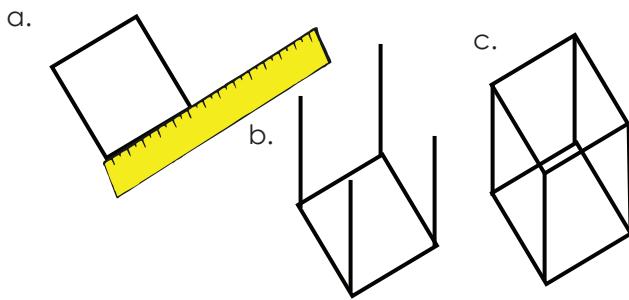
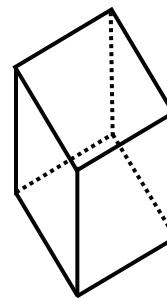


5. Draw by following the steps.

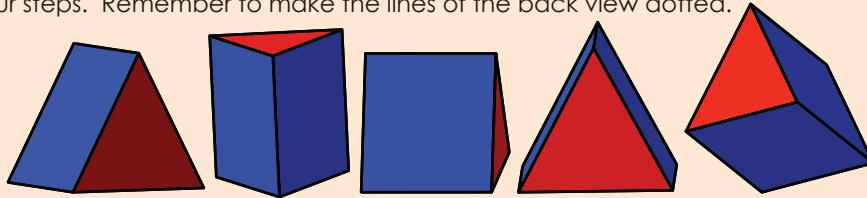


Step 3 Remove the cube.**Step 4**

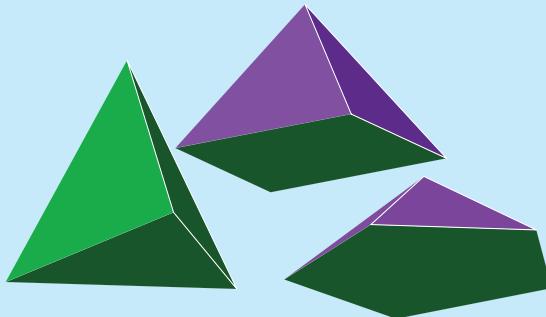
Measure your angle to see how close you were.

**Step 5** a. Measure the length of the sides.
b. Draw lines showing the height of the cube of the same length.
c. Draw the top of the cube.**Step 6** It is important to use dotted lines to show the back of the cube (or any other geometric solid).**Problem solving**

Sit at your desk, look at the sketches in your book and then place the geometric solid in the same position on your desk. Are all of the drawings possible? Make a drawing of any of these drawings showing it in four steps. Remember to make the lines of the back view dotted.



What is a pyramid? Look at the pictures and describe a pyramid.



Where do we find real pyramids?

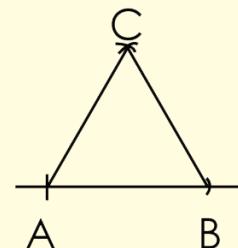
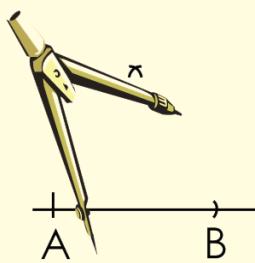


Will we only find pyramids in Egypt?

1. Construct the net for a tetrahedron.

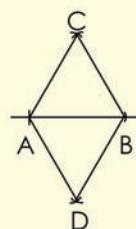
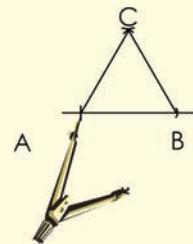
Step 1:

Construct an equilateral triangle. Label it ABC.



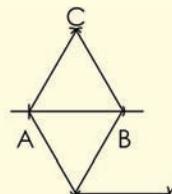
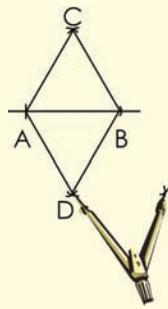
Step 2:

Construct another equilateral triangle with one base joined to base AB of the first triangle.



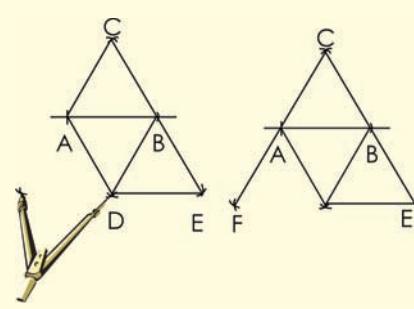
Step 3:

Construct another triangle using BD as a base.



Step 4:

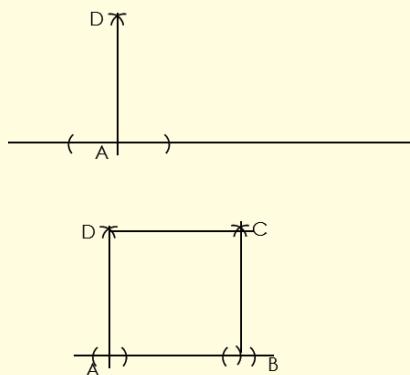
Construct another triangle using AD as a base.



2. Construct a square pyramid net.

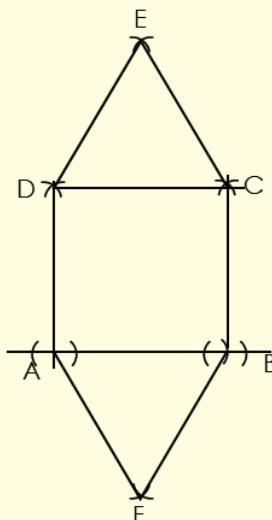
Step 1:

Construct two perpendicular lines. The lengths of AD and AB should be the same. Use your pair of compasses to measure them. From there, construct rectangle ABCD.



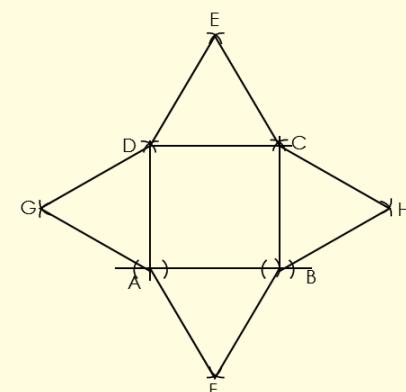
Step 2:

- Using AB as a base, construct a triangle.
- Using DC as a base, construct a triangle.



Step 3:

- Using DA as a base, construct a triangle.
- Using BC as a base, construct a triangle.



i) After you have constructed the square-based pyramid, answer the following questions:

- what difficulties did you have?

- what would you do differently next time?

ii) Now do the construction on cardboard, cut it out and make the square pyramid.

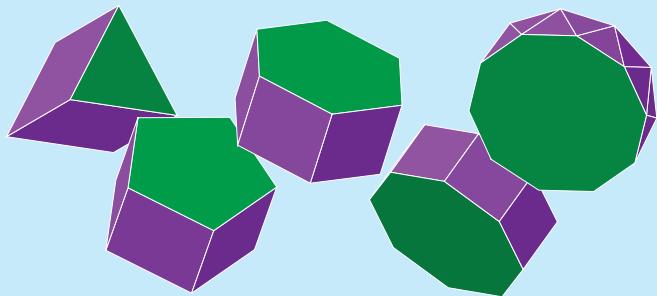


Problem solving

Look at this gift box and make it yourself.



What is a prism? Look at the pictures and describe a prism

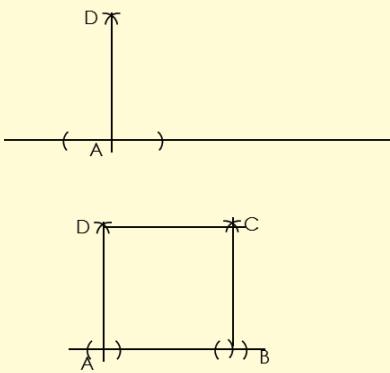


Sometimes people think a prism only takes on this shape. How will you find out if this is true?

1. Construct the net of a triangular prism.

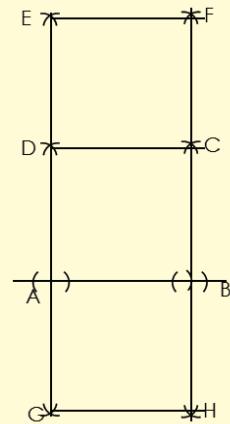
Step 1:

Construct two perpendicular lines. The lengths of AD and AB could be the same or one longer to form a rectangle. Use your pair of compasses to measure them). From there, construct rectangle ABCD.



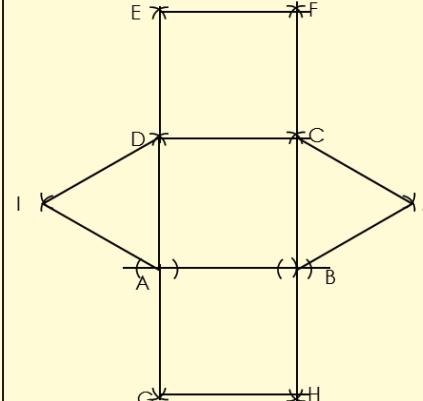
Step 2:

- Using DC as a base, construct a square (or rectangle).
- Using AB as a base, construct another square (or rectangle).



Step 3:

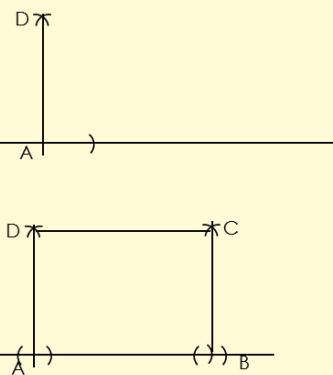
- Using DA as a base, construct a triangle.
- Using BC as a base, construct a triangle.



2. Rectangular prism construction.

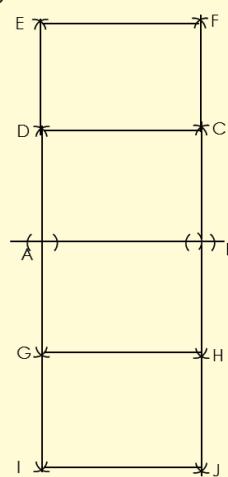
Step 1:

Construct two perpendicular lines. The length between A and B should be longer than that between D and A. Use your compass to measure them. From there, construct rectangle ABCD.



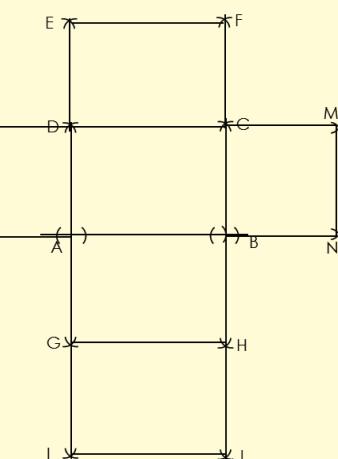
Step 2:

- Use DC as base to construct another rectangle above.
- Use AB as base to construct another rectangle below. Label the new points G and H.
- Use GH as base to construct another rectangle.



Step 3:

- Use DA as base to construct a square.
- Use CB as base to construct a square.

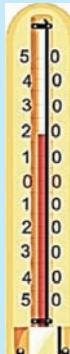


What is this prism showing us?



Problem solving





"What is the temperature on a hot, sunny day?"

Point out the degrees on this thermometer. What does it mean for the temperature to be two degrees below zero? Show where this is on the thermometer.

You would use a negative sign to write this number since it is below zero.

-2

Where is the five degrees below zero on the thermometer? Is this hotter or colder than two degrees below zero?



If you turn the thermometer sideways it becomes like a number line and shows that the negative numbers are to the left of zero and positive numbers are to the right of zero, with zero being neither positive nor negative.

1. Write the appropriate temperature for the given weather condition.

- What would the temperature be on a hot and sunny day? _____
- What would the temperature be on a cool spring day? _____
- What would the temperature be on a frosty winter morning? _____
- Write the temperature of eight below zero. _____
- Which is colder, eight below zero or 10 below zero? Why? _____
- Draw a thermometer and label where 10 below zero would be.

2. Write where the money in each statement will go, in the negative or positive column.

Statement	Positive	Negative
a. Peter won R100 in the draw.		
b. The weather report said that in Sutherland it is going to be seven degrees below zero.		
c. Sindy lost her purse with R20 in it.		

d. David sold his cell phone for R200.		
e. I bought airtime for R50 with some of my savings.		
f. We raised R500 during the course of the day		
g. We used R100 from the money raised to buy snacks for the party.		
h. My older brother earned R120 for the work he had done		
i. We made R300 profit.		
j. We made a R200 loss.		

3. Complete the questions below after completing the table in Question 2.

a. Circle the key word in each sentence that helped you to make the decision.

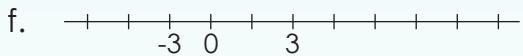
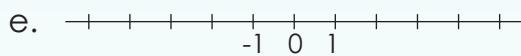
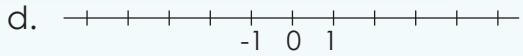
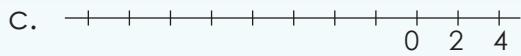
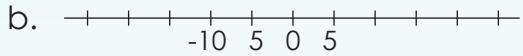
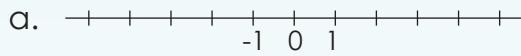
b. What characteristics are found in the positive column? _____

c. What characteristics are found in the negative column? _____

d. Write down all the characteristics of integers. _____

e. Where are integers used in everyday life? Give examples of your own or cut examples from a newspaper.

4. Complete these number lines.



5. Complete the following

a. $\{3, 2, 1, 0, \underline{\quad}, \underline{\quad}, \underline{\quad}\}$

b. $\{-10, -9, -8, \underline{\quad}, \underline{\quad}, \underline{\quad}\}$

c. $\{8, 6, 4, 2, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}\}$

d. $\{-9, -6, -3, \underline{\quad}, \underline{\quad}, \underline{\quad}\}$

e. $\{12, 8, 4, \underline{\quad}, \underline{\quad}, \underline{\quad}\}$

Problem solving

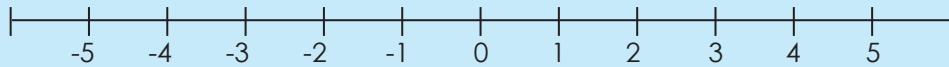
Take a newspaper and find five negative numbers in it.

a. Explain what each number tells us.

b. Write down the opposite numbers for the five numbers



More integers



- What do we call the units to the right of the zero?
- What do we call the units to the left of the zero?
- What will five units left from 3 be?
- What will five units right from 3 be?
- What is the opposite of -4 ?
- What is the opposite of 4 ?
- What is three below zero?

(positive numbers or integers)

(negative numbers or integers)

1. Write an integer to represent each description.

- Five units to the left of 4 on a number line. _____
- 20 below zero. _____
- The opposite of 271. _____
- Eight units to the left of -3 on a number line. _____
- Eight units to the right of -3 on a number line. _____
- 16 above zero. _____
- 14 units to the right of -2 on a number line. _____
- Seven units to the left of -8 on a number line. _____
- The opposite of -108 . _____
- 15 below zero. _____

2. Order these integers from smallest to biggest.

- $-5, -51, 21, -61, 42, -66, 5, 39, -31, -71, 31, 66$

- $42, 21, 48, 72, -64, -20$

- $15, -30, -14, -3, 9, 31, 21, 26, 4, -31, -24, 44$

d. $-41, 54, -31, -79, 57$

e. $-26, 32, 23, 10, -31, 12, 31, 26$

f. $43, -54, 44, -55, -37, 22, 52, -39, -43, -56, 18$

g. $-41, -23, -31, 40, -21, 2$

h. $4, -10, 15, 7, 10, -2, -13, -6, -12, 9, 12$

i. $-7, -15, -25, -24, -12, -13, 22, 6, 11, 2$

j. $73, -24, -20, 21, -44, 5, -2, 41, 55$

3. Fill in $<$, $>$ or $=$

a. $-2 \quad \square \quad 2$

b. $-10 \quad \square \quad 10$

c. $-5 \quad \square \quad 0$

d. $-4 \quad \square \quad -3$

e. $-9 \quad \square \quad -6$

f. $-20 \quad \square \quad -16$

4. Give five numbers smaller than and bigger than:

a. -2

b. -99

c. 1

Smaller	Bigger

Smaller	Bigger

Smaller	Bigger

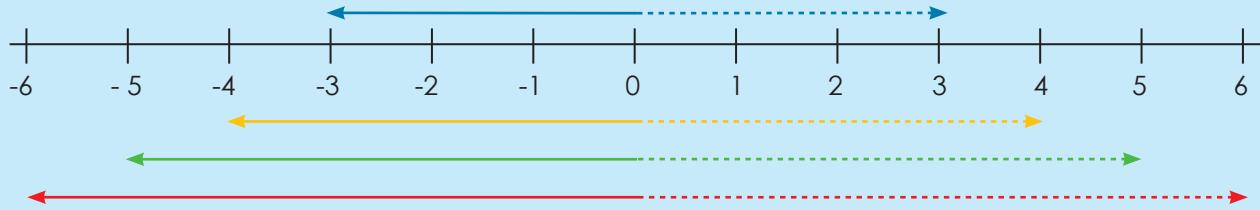
Problem solving

Make your own word problem using a negative and a positive number



Calculate integers

What is the opposite of -3 ? How many units will it be from -3 to 3 ?



Explain the lines above.

1. We have learnt that two integers are opposites if they are each the same distance away from zero. Write down the opposite integers for the following:

a. -2 _____

b. 3 _____

c. -7 _____

d. 8 _____

e. -10 _____

f. -15 _____

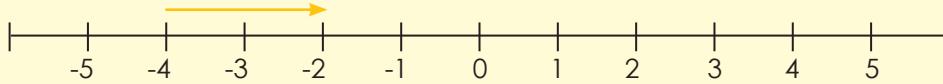
g. 1 _____

h. -100 _____

i. 75 _____

2. Calculate the following.

Example: $-4 + 2 = -2$



a. $-5 + 5 =$

b. $-2 + 3 =$

c. $-7 + 8 =$

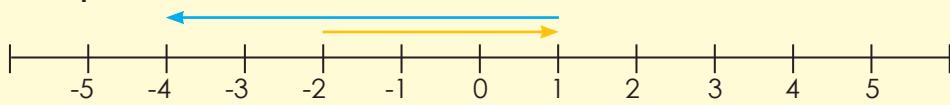
d. $-2 + 3 =$

e. $+4 - 6 =$

f. $10 - 12 =$

3. Calculate the following.

Example: $-2 + 3 - 5 = -4$



a. $-3 + 2 - 5 =$

b. $2 - 6 + 10 =$

c. $-6 + 8 - 7 =$

d. $-3 + 10 - 11 =$

e. $9 - 11 + 2 =$

f. $2 - 8 + 7 =$

4. Complete the following.

Example: Subtract 7 from -2. Count backwards: -2, -3, -4, -5, -6, -7, -8, -9

Add 2 to -5. Count forwards: -5, -4, -3

a. Subtract 4 from -3 _____

b. Subtract 6 from -8 _____

c. Subtract 5 from 3 _____

d. Subtract 9 from 7 _____

e. Subtract 3 from -2 _____

Problem solving

What is:

The sum of 10 and 8, and the sum of -9 and -8?

The sum of 101 and 85, and the sum of -98 and -104?

The sum of 19 and -8, and the sum of -19 and 8?

The sum of -7 and -14, and the sum of -4 and 20?

The sum of 100 and -50, and the sum of -100 and 50?



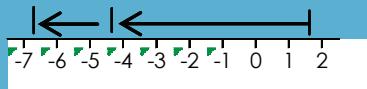
Discuss the following

Add integers with the same sign

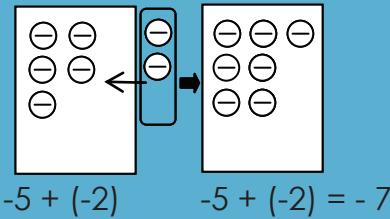
Find $-5 + (-2)$.

Method 1: Use a number line.

- Start at zero.
- Move 5 units left.
- From there, move 2 units left.



Method 2: Draw a diagram.

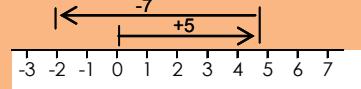


Add integers with different signs

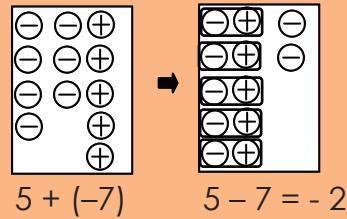
Find $5 + (-7)$.

Method 1: Use a number line.

- Start at zero.
- Move 5 units right.
- From there, move 7 units left.



Method 2: Draw a diagram.



1. Complete the following.

- Number line method
- Drawing a diagram

a. Find $-8 + (-3)$

c. Find $-4 + (-5)$

e. Find $-18 + (-7)$

b. Find $-12 + (-8)$

d. Find $-7 + (-9)$

f. Find $6 + (-8)$

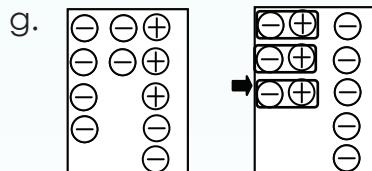
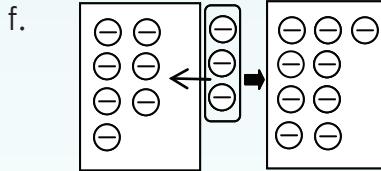
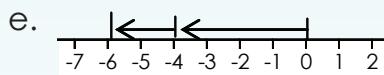
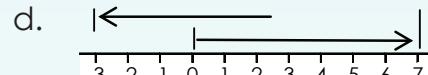
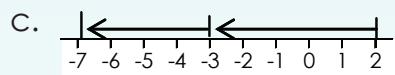
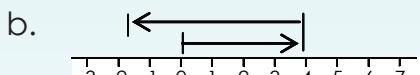
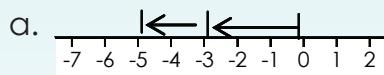
g. Find $9 + (-11)$

h. Find $6 + (-9)$

i. Find $3 + (-16)$

j. Find $8 + (-19)$

2. Write sums for the following.



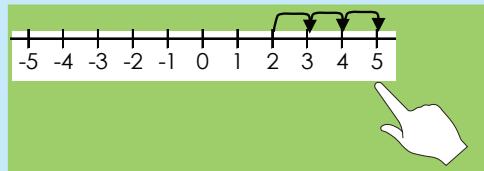
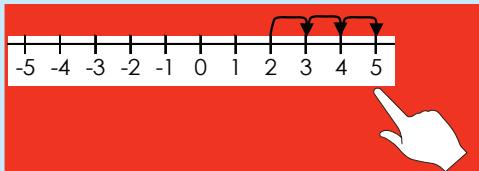
Help-a-friend!

Write down step-by-step how you will explain integers to a friend that missed one day at school.

Adding and subtracting integers

Adding a negative number is just like subtracting a positive number: $2 + -3 = 2 - 3$

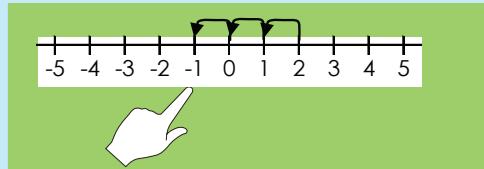
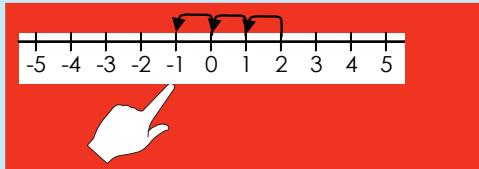
If you are **adding a positive number**, move your finger to the **right** as many places as the value of that number. For example, if you are adding 3, move your finger three places to the right: $2 + 3 = 5$



If you are **subtracting a negative number**, move your finger to the **right** as many places as the value of that number. For example, if you are subtracting -3 , move your finger three places to the right: $2 - -3 = 5$

Subtracting a negative number is just like adding a positive number. The two negatives cancel each other out. $2 + 3 = 2 - -3$

If you are **adding a negative number**, move your finger to the **left** as many places as the value of that number. For example, if you are adding -3 , move your finger three places to the left: $2 + -3 = -1$



If you are **subtracting a positive number**, move your finger to the **left** as many places as the value of that number. For example, if you are subtracting 3 , move your finger three places to the left: $2 - 3 = -1$

1. Calculate the following, make use of the number lines.

a. $4 + -5 =$

b. $5 + -7 =$

c. $5 + -7 =$

d. $6 + -9 =$

e. $3 + -2 =$

f. $4 + -7 =$

2. Calculate the following:

a. $4 - -5 =$ _____

b. $5 - -7 =$ _____

c. $5 - -7 =$ _____

d. $6 - -9 =$ _____

e. $3 - -2 =$ _____

f. $4 - -7 =$ _____

g. $5 - -4 =$ _____

h. $2 - -1 =$ _____

i. $3 - -4 =$ _____

j. $1 - -3 =$ _____

k. $2 - -5 =$ _____

l. $5 - -11 =$ _____

m. $7 - -6 =$ _____

n. $8 - -12 =$ _____

o. $5 - -9 =$ _____

p. $4 - -4 =$ _____

q. $3 - -3 =$ _____

r. $5 - -12 =$ _____

s. $2 - -4 =$ _____

t. $3 - -6 =$ _____

u. $5 - -6 =$ _____

v. $3 - -8 =$ _____

w. $7 - -10 =$ _____

x. $6 - -6 =$ _____

y. $4 - -6 =$ _____

z. $7 - -14 =$ _____

3. Explain in your own words what you had to do to get to the answer:

a. In number 1.

b. In number 2.

Problem solving

Make your own problem using integers.



Integer calculations

Describe:

Positive number

+

Negative number

=

Positive answer
Negative answer

Positive number

-

Negative number

=

Positive answer
Negative answer

Negative number

+

Positive number

=

Positive answer
Negative answer

Negative number

-

Positive number

=

Positive answer
Negative answer

1. Calculate the following:

a. $12 + -31 =$

b. $-28 + -42 =$

c. $7 + -34 =$

d. $33 + -44 =$

e. $5 + -432 =$

f. $-15 + -20 =$

g. $-15 + 5 =$

h. $19 + 14 =$

i. $25 + 4 =$

j. $4 + 7 =$

2. Calculate the following.

Example: $-14 - -20$
 $= -14 + 20$
 $= 6$

a. $7 - -31 =$

b. $35 - 31 =$

c. $-17 - 8 =$

d. $47 - -46 =$

e. $-41 - 17 =$

f. $28 - -46 =$

g. $-47 - -7 =$

h. $-28 - 15 =$

i. $-15 - 3 =$

j. $5 - 31 =$

3. Calculate the following:

a. $\underline{\hspace{1cm}} + 44 = 42$

b. $\underline{\hspace{1cm}} + -18 = -32$

c. $\underline{\hspace{1cm}} + -21 = -30$

d. $-3 + \underline{\hspace{1cm}} = 33$

e. $14 + \underline{\hspace{1cm}} = 16$

f. $14 + \underline{\hspace{1cm}} = 63$

g. $42 + \underline{\hspace{1cm}} = 65$

h. $\underline{\hspace{1cm}} + -10 = -12$

i. $38 + \underline{\hspace{1cm}} = 65$

j. $-46 + \underline{\hspace{1cm}} = -72$

k. $\underline{\hspace{1cm}} + -43 = -41$

l. $\underline{\hspace{1cm}} + -16 = 30$

m. $\underline{\hspace{1cm}} + -44 = -81$

n. $\underline{\hspace{1cm}} + -31 = 6$

o. $\underline{\hspace{1cm}} + -28 = -32$

p. $11 + \underline{\hspace{1cm}} = -19$

q. $\underline{\hspace{1cm}} + 24 = 6$

r. $45 + \underline{\hspace{1cm}} = 73$

s. $\underline{\hspace{1cm}} + -29 = 1$

t. $12 + \underline{\hspace{1cm}} = -32$

u. $-44 + \underline{\hspace{1cm}} = -15$

v. $\underline{\hspace{1cm}} + 24 = -11$

w. $\underline{\hspace{1cm}} + 10 = 33$

x. $\underline{\hspace{1cm}} + 49 = 18$

y. $\underline{\hspace{1cm}} + 4 = 26$

z. $41 + \underline{\hspace{1cm}} = 60$

Problem solving

Give three integers of which the sum is -9. Use two positive integers and one negative integer

Give three integers of which the sum is -4. Use two negative integers and one positive integer.

Give four integers of which the sum is -11. Use two negative integers and two positive integers.



commutative property integers

The commutative property of number says that you can swap numbers around and still get the same answer.

This is when you add or multiply.



$$8 + 4 = 4 + 8$$

$$5 \times 4 = 4 \times 5$$

In this worksheet we will work with integers.



1. Use the commutative property to make the equation equal. Calculate it.

Example: $8 + (-3) = (-3) + 8 = 5$
 $(-8) + 3 = 3 + (-8) = -5$

a. $4 + (-5)$

b. $(-10) + 7$

c. $3 + (-9)$

d. $8 + (-11)$

e. $(-4) + 8$

f. $9 + (-2)$

2. Substitute and calculate.

Example: $a = -2$ and $b = 3$
 $a + b = b + a$
 $(-2) + 3 = 3 + (-2)$
 $1 = 1$

a. $a + b = b + a$ if $a + 4; b = -1$

b. $a + b = b + a$ if $a + -2; b = 7$

c. $a + b = b + a$ if $a = -2$; $b = 7$

d. $x + y = y + x$ if $x = -1$; $y = 13$

e. $x + y = y + x$ if $x = -5$; $y = 9$

f. $d + e = e + d$ if $e = -12$; $d = 7$

g. $t + s = s + t$ if $t = -4$; $s = 10$

h. $a + b = b + a$ if $a = -10$; $b = 7$

i. $y + z = z + y$ if $z = -8$; $y = 2$

j. $k + m = m + k$ if $k = -13$; $m = 20$

Problem solving

Use the commutative property to make your own equation and prove that it is equal using the numbers -8 and 21 .





Associative property and integers

The Associative property of number means that it doesn't matter how you **group the numbers** when you **add** or when you multiply.



So, in other words it doesn't matter which you calculate first.



Example addition:

$$(2 + 3) + 5 = 2 + (3 + 5)$$

Because $5 + 5 = 2 + 8 = 10$

Example multiplication:

$$(2 \times 4) \times 3 = 2 \times (4 \times 3)$$

$$8 \times 3 = 2 \times 12 = 24$$

In this worksheet we will look at integers and associative property

1. Use the associative property to calculate the following.

Example: $[(2 + 3) + (-4)] = 2 + [3 + (-4)]$
 $5 - 4 = 2 - 1$
 $1 = 1$

$$[(-2) + (3 + 4)] = [(-2 + 3) + 4]$$

 $-2 + 7 = 1 + 4$
 $5 = 5$

$$[(-3) + (2 + 4)] = [(-3 + 2) + 4]$$

 $-3 + 6 = -1 + 4$
 $3 = 3$

a. $[(-6) + (4 + 2)]$

b. $[3 + 7 + (-5)]$

c. $[(6 + 4) + (-2)]$

d. $[(-3) + 7 + 5]$

e. $[(-4) + (6 + 2)]$

f. $[3 + (-7) + 5]$

g. $[(-9) + (3 + 11)]$

h. $[(12 + 13) + (-10)]$

i. $[(-3) + (9 + 11)]$

j. $[(-12) + (13 + 10)]$

2. Substitute and calculate.

Example: $a = -7, b = 1, c = 2$

$$(a + b) + c = a + (b + c)$$

$$[(-7) + 1] + 2 = (-7) + (1 + 2)$$

$$-6 + 2 = -7 + 3$$

$$-4 = -4$$

a. $(a + b) + c = a + (b + c)$

If: $a = 4$

$$b = -5$$

$$c = 3$$

b. $(a + b) + c = a + (b + c)$

If: $a = 2$

$$b = 9$$

$$c = -4$$

c. $a + (b + c) = (a + b) + c$

If: $a = -8$

$$b = 1$$

$$c = 2$$

d. $a + (b + c) = (a + b) + c$

If: $a = -2$

$$b = 11$$

$$c = 12$$

Problem solving

Use the associative property to make your own equation and prove that it is equal using the numbers -5, 17 and 12.





Integers: distributive property and integers

The Distributive property of number says you get the same answer when you ... I cannot remember please help me.



$$4 \times (2 + 5) = (4 \times 2) + (4 \times 5)$$

...multiply a number by a group of numbers added together as when you do when you multiply each number separately and then add the products.



Oh! So the $4 \times$ can be **distributed** across the $2 + 5$.



In this worksheet we will work with integers.

1. Use the distributive property to calculate the sums. Before you calculate highlight or underline the distributed number.

Example: $-2 \times (3 + 4) = (-2 \times 3) + (-2 \times 4)$
 $-2 \times 7 = -6 + -8$
 $-14 = -14$

$$2 \times (-3 + 4) = (2 \times -3) + (2 \times 4)$$
$$2 \times 1 = -6 + 8$$
$$2 = 2$$

$$2 \times (3 + -4) = (2 \times 3) + (2 \times -4)$$
$$2 \times (-1) = 6 + -8$$
$$-2 = -2$$

a. $-4 \times (2 + 1)$

b. $-5 \times (3 + 6)$

c. $4 \times (-2 + 1)$

d. $5 \times (-3 + 6)$

e. $4 \times (2 + -1)$

f. $5 \times (3 + -6)$

g. $(-3 \times 2) + (-3 \times 4)$

h. $(-7 \times 1) + (-7 \times 4)$

i. $(8 \times -4) + (8 \times 2)$

2. Substitute and calculate.

Example: $a \times (b + c)$ if $a = -4$, $b = 3$, $c = 1$
 $a \times (b + c) = (a \times b) + (a \times c)$
 $-4 \times (3 + 1) = (-4 \times 3) + (-4 \times 1)$
 $-4 \times 4 = -12 + -4$
 $-16 = -16$

a. $a \times (b + c)$
if $a = 2$, $b = -3$, $c = -5$

b. $a \times (b + c)$
if $a = -7$, $b = 2$, $c = 3$

c. $a \times (b + c)$
if $a = 1$, $b = -8$, $c = 2$

d. $(a \times b) + a + c$
if $a = 3$, $b = -10$, $c = 5$

e. $m \times (n + p)$
if $m = 3$, $n = 2$, $p = -11$

f. $(m \times n) + (m \times p)$
if $m = 7$, $n = 8$, $p = -9$

Problem solving

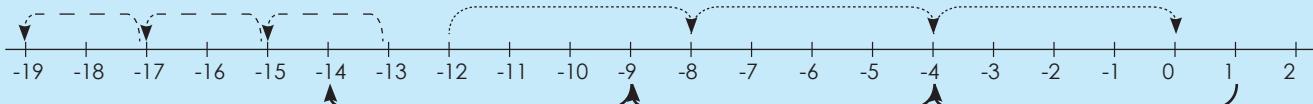
Make use of the distributive property to write your own equation for:

$a = -4$, $b = 5$ and $c = 11$



Number patterns: constant difference and ratio

Describe the patterns using "adding" and "subtracting".



..... Adding 2: -19, -17, -15, -13

..... Subtracting 4: 0, -4, -8, -12

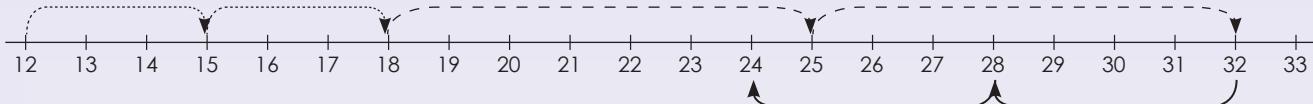
..... Adding 5: -14, -9, -4, 1

1. Describe each pattern.

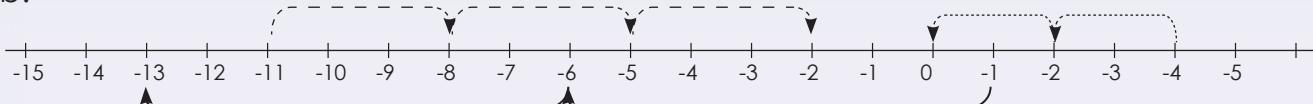
Describe the pattern in your own words.



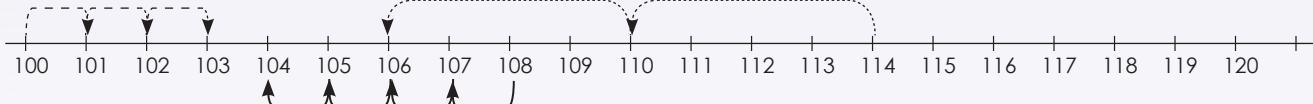
a.



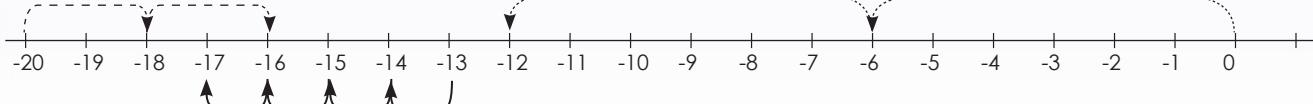
b.



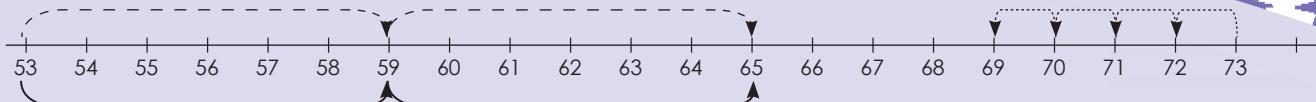
c.



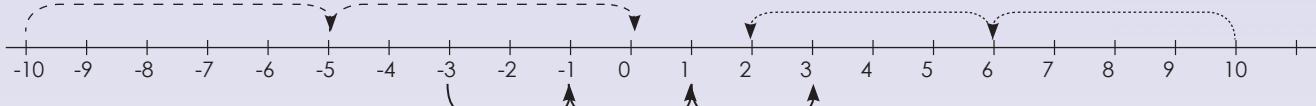
d.



e.



f.



2. Describe the pattern.

Example: $-12, -8, -4, 0$

Adding 4

a. $16, 11, 6, 1$

b. $25, 22, 19, 16$

c. $-16, -8, 0, 8$

d. $-4, -1, 2, 5$

e. $-79, -69, -59, -49$

f. $58, 50, 42, 34$

3. Describe the pattern.

Example: $-12 \times 4 = -48 \times 4 = -192 \times 4 = -768$

Multiplying the previous number by 4

a. $7, -21, 63, -189$

b. $-4, -44, -484, -5324$

c. $-11, -66, -396, -2376$

d. $2, -8, 32, -128$

e. $9, 72, 576, 4608$

f. $-5, -45, -405, -3645$

Problem solving

Brenda collects shells. Every day she picks up double the amount of the previous day. On day 1 she picks up 8 shells. On day 2 she collects 16. How many shells would she pick up on day 3 if the pattern continued? Write down the rule.



Number patterns: neither a constant difference nor a constant ratio

Describe the following: -1, -2, -4, -7, -11, -16, ...

What will the next three terms be, using the identified rule?

Take your time to describe the pattern in words.

This pattern has neither a constant difference nor a constant ratio. It can be described in your own words as "increasing the difference between consecutive terms by 1 each time" or "subtracting 1 more than what was subtracted to get the previous term". Using this rule, the next three terms will be -22, -29, -37.

1. Describe the pattern and make a drawing to show each term.

Example: 15, 22, 16, 21, 17

15 + 7 22 - 6 16 + 5 21 - 4 17

Each number of the number pattern is called a term.

a. -4, 1, 5, 8, 10

b. 8, 10, 13, 17, 22

c. 2, -2, -8, -16, -26

d. -11, -12, -10, -13, -9

e. -7, -1, 11, 29, 53

f. 5, -3, -10, -16, -21

2. What will the tenth pattern be?

Example: 12, 24, 36, 48

Position of the term \times 12

Position in the sequence	1	2	3	4		10
Term	12	24	36	48		120

a.	Position in the sequence	1	2	3	4		10
	Term	-5	-10	-15	-20		

b.	Position in the sequence	1	2	3	4		10
	Term	8	16	24	32		

c.	Position in the sequence	1	2	3	4		10
	Term	-12	-24	-36	-48		

d.

Position in the sequence	1	2	3	4		10
Term	7	14	21	28		

e.

Position in the sequence	1	2	3	4		10
Term	-11	-22	-33	-44		

3. What will the term be?

Example: 1, 4, 9, 16

Position of the term squared

Position in the sequence	1	2	3	4		15
Term	1	4	9	16		225

a.

Position in the sequence	1	2	3	4		20
Term	25	50	75	100		

b.

Position in the sequence	1	2	3	4		104
Term	-4	-8	-12	-16		

c.

Position in the sequence	1	2	3	4		59
Term	1	8	27	64		

d.

Position in the sequence	1	2	3	4		36
Term	13	26	39	52		

e.

Position in the sequence	1	2	3	4		29
Term	21	42	63	84		

Problem solving

Thabo builds a brick wall around the perimeter of his house. On the first day he uses 75 bricks, on the second day he uses 125 and on the third day he uses 175. How many bricks will he need on the fourth day? Write a rule for the pattern.

Ravi draws 2 figures on the first page, 4 figures on the second page, 8 figures on the third page, and 16 figures on the fourth page. If this pattern continued, how many figures would Ravi draw on the fifth page?

Lisa read 56 pages on Sunday, 66 pages on Monday, 76 pages on Tuesday, and 86 pages on Wednesday. If this pattern continued, how many pages would Lisa read on Thursday?

Thandi cut 1 rose from the first plant, 3 roses from the second plant, 7 roses from the third plant, and 13 roses from the fourth plant. If this pattern continued, how many rose would Thandi cut from the fifth plant?





Number sentences and words

Describe the relationships between the numbers in a sequence.

-4, -7, -10, -13, ...

Identify the:

First term: -4

Second term: -7

Third term: -10

Fourth term: -13

What will the 5th term be?



What are the rules for the sequences: ("subtracting 3")

First term: $-4 = -3(1) - 1$

Second term: $-7 = -3(2) - 1$

Third term: $-10 = -3(3) - 1$

Fourth term: $-13 = -3(4) - 1$

The number in the brackets corresponds to the position of the term in the sequence.

If the number in the brackets represents the term, what will the 20th term be?

1. Look at the following sequences:

- Calculate the 20th term using a number sentence.
- Describe the rule in your own words.

Example: Number sentence: -6, -10, -14, -18

Rule in words: $(-4 \times \text{the position of the term}) - 2$.

a. Number sentence: 8, 14, 20, 26

i.

ii.

b. Number sentence: 0, -3, -6, -9

i.

ii.

c. Number sentence: -4, -5, -6, -7

i.

ii.

d. Number sentence: -2, 3, 8, 13

i.

ii.

e. Number sentence: -2, -6, -10, -14

i.

ii.

f. Number sentence: -1, 6, 13, 21

i.

ii.

g. Number sentence: 13, 21, 29, 37

i.

ii.

h. Number sentence: 0, 1, 2, 3

i.

ii.

i. Number sentence: 7, 5, 3, 1

i.

ii.

j. Number sentence: 2, 4, 6, 8

i.

ii.

Problem solving

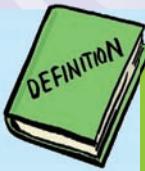
Tshepo earns R25 per week for washing his father's motor car. If he saves R5,50 the first week, R7,50 the second week and R9,50 the third week, how much would he save in the fourth week if the pattern continued?

Calculate the total amount he saved an over 4 weeks. Write a rule for the number sequence.





Number sequences: describe a pattern



A sequence is a list of numbers or objects which are in a special order.

E.g.

Number sequence: -2, -4, -6, -8

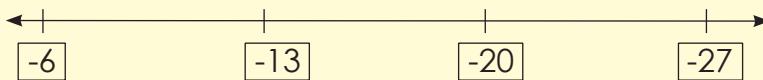
Geometric sequence: -2, -4, -8, -16

What is the difference between a number sequence and a geometric sequence?
Give one example of each.

1. Describe the sequence in different ways using the template provided.

Example: -6, -13, -20, -27

i) Write it on a number line.



ii) Write it in a table.

Position in the sequence	1	2	3	4
Term	-6	-13	-20	-27
	$-7(1)-1$	$-7(2)-1$	$-7(3)-1$	$-7(4)-1$

iii) Where n is the position of the term.

$$\text{First term: } -7(1)-1 = -6$$

$$\text{Second term: } -7(2)-1 = -13$$

$$\text{Third term: } -7(3)-1 = -20$$

$$\text{Fourth term: } -7(4)-1 = -27$$

$$\text{n term: } 7(n)-1$$

a. -1, 2, 5, 8



ii)

Position in the sequence	1	2	3	4
Term				

Below the table, four empty boxes are aligned horizontally, each with a small arrow pointing towards it from below, corresponding to the positions 1, 2, 3, and 4.

iii) Where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

n term:

b. 3, 5, 7, 9



ii)

Position in the sequence	1	2	3	4
Term				

Below the table, four empty boxes are aligned horizontally, each with a small arrow pointing towards it from below, corresponding to the positions 1, 2, 3, and 4.

iii) Where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

n term:

continued



Number sequences: describe a pattern continued

c. -11, -19, -27, -35



ii)

Position in the sequence	1	2	3	4
Term				

Below the table, four empty boxes are aligned horizontally, each with a small arrow pointing towards the right side of the table. The first three boxes are under the 'Term' column headers, and the fourth box is under the empty fourth column.

iii) Where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

n term:

d. 16, 22, 28, 34



ii)

Position in the sequence	1	2	3	4
Term				

Below the table, four empty boxes are aligned horizontally, each with a small arrow pointing towards the right side of the table. The first three boxes are under the 'Term' column headers, and the fourth box is under the empty fourth column.

iii) Where n is the position of the term.

First term:

Second term:

n term:

Third term:

Fourth term:

e. $-4, -9, -14, -19$

i)

ii)

Position in the sequence	1	2	3	4
Term	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

iii) Where n is the position of the term.

First term:

Second term:

n term:

Third term:

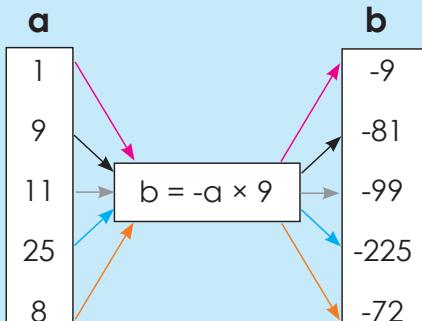
Fourth term:

Problem solving

Write the rule for the number sequence: $-3, -5, -7, -9$



Look and discuss.

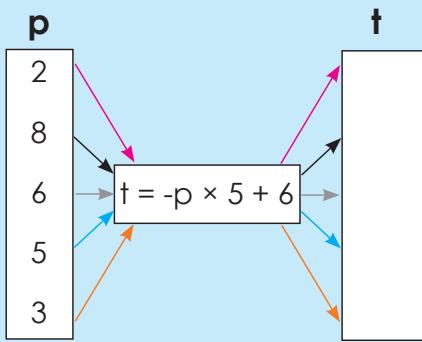


The rule is: $b = -a \times 9$

$b = -a \times 9$. Look at the flow diagram.

Which numbers can replace a ?

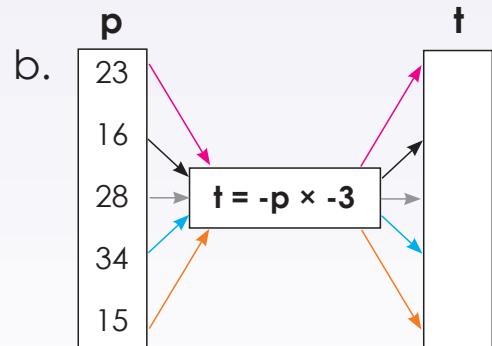
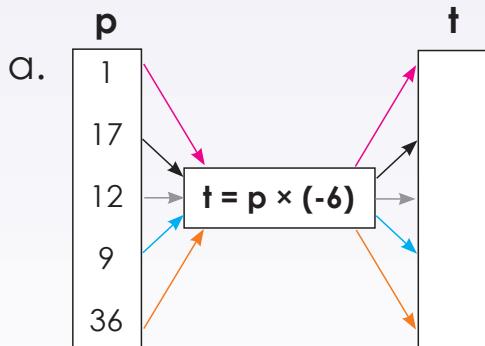
- $b = -1 \times 9 = -9$
- $b = -9 \times 9 = -81$
- $b = -11 \times 9 = -99$
- $b = -25 \times 9 = -225$
- $b = -8 \times 9 = -72$

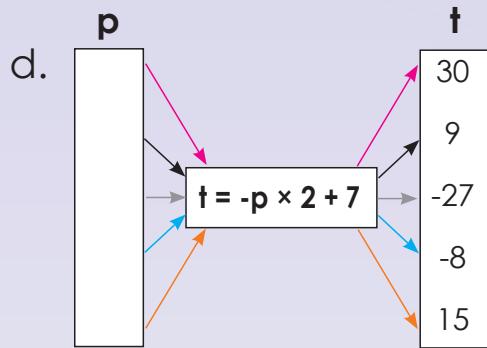
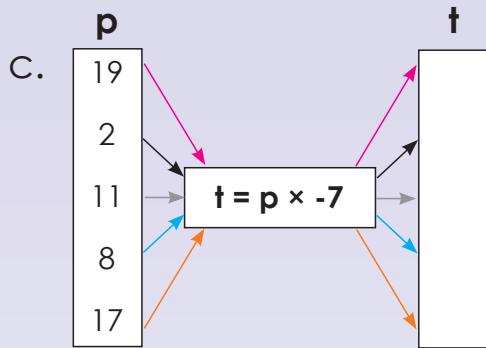


Calculate:

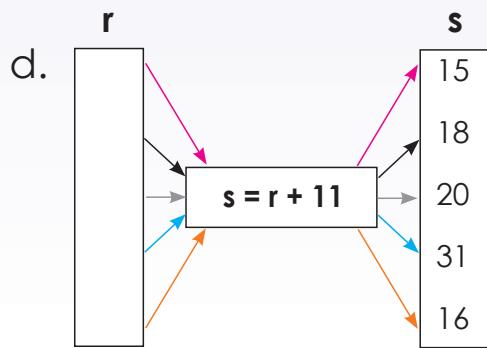
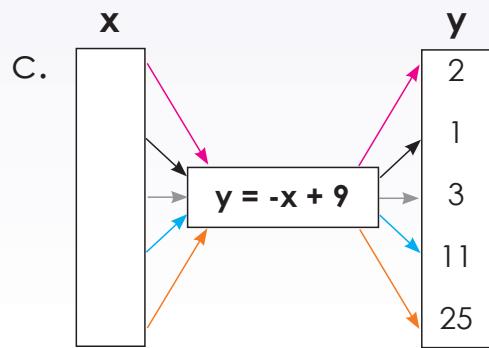
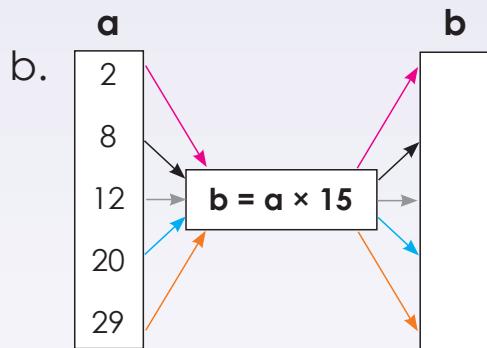
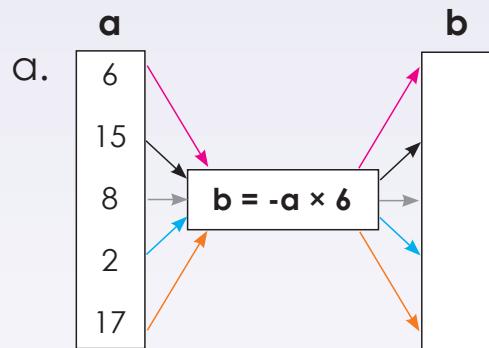
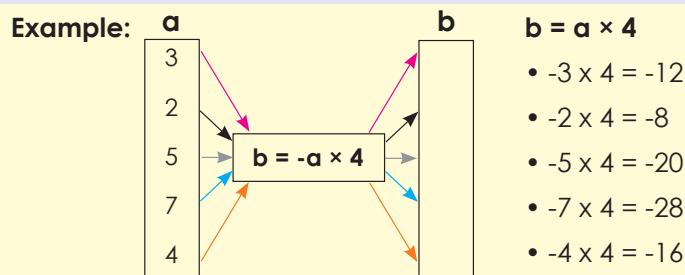
- $t = -2 \times 5 + 6 = -16$
- $t = -8 \times 5 + 6 = -46$
- $t = -6 \times 5 + 6 = -36$
- $t = -5 \times 5 + 6 = -31$
- $t = -3 \times 5 + 6 = -21$

1. Revision: complete the flow diagrams.

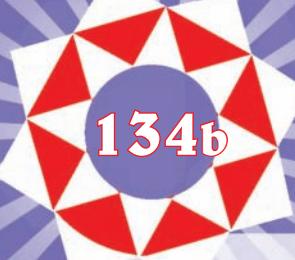




2. Use the given rule to calculate the value of b .



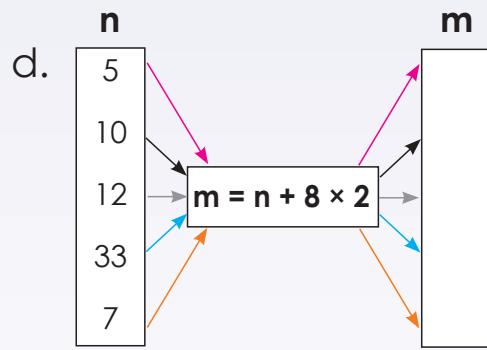
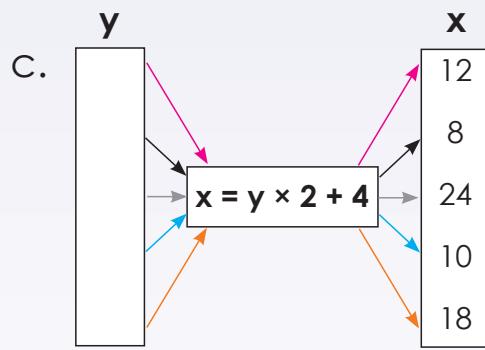
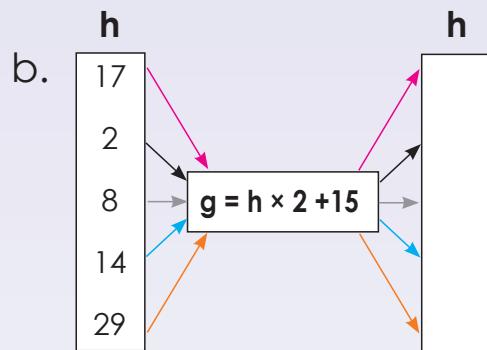
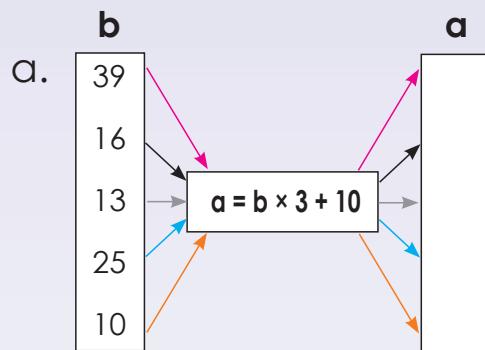
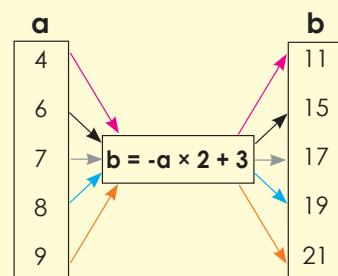
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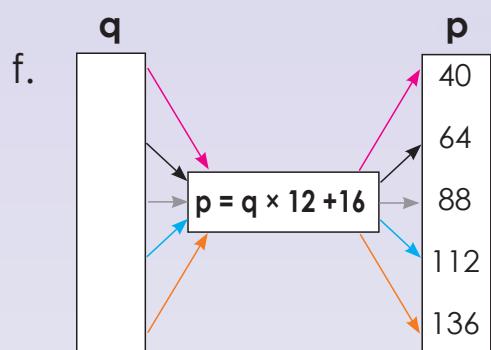
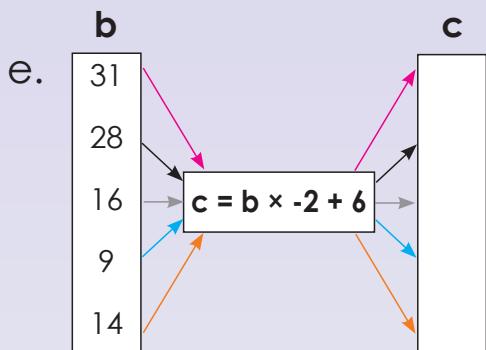


Input and output values continued

3. Use the given rule to calculate the variable.

Example:





4. Prepare one flow diagram to present to the class.

Problem solving

- Draw your own spider diagram where $a = -c - 9$.
- Draw your own spider diagram where $a = c \times 3 - 7$



More input and output values

x	1	2	3	4		12	n
y	5	7	9	11		m	93

The rule $y = 2x + 3$ describes the relationship between the given x and y values in the table.

45

Why is $n = 45$ and $m = 27$?

27

To find m and n , you have to substitute the corresponding values for x or y into the rule and solve the equation by inspection.

Multiple rules are acceptable if they match the given input values to the corresponding output values

Now try and find another rule

1. Solve for m and n

a. $x = 3y - 1$

y	2	4	6	n	10	20
x				23		m

b. $x = -2y + 6$

y	1	2	3		5	n
x				m		-174

c. $y = -4x - 2$

x	3	4	5	6	n	10	100
y					-30		m

d. $y = x + 2$

x	2	n	4	5	16	17
y		5			m	

e. $t = -8s + 2$

s	1	2	3	n	5	6	7
t				30	m		

f. $p = 7q - 7$

f	1	5	10	20	n	100
q				m		168

1. What is the value of m and n ?

Example: $y = -7x + 2$

x	1	2	3	4		15	m	60
y	-5	-12	-19	-26		-103	18	n

Rule: the given term $x - 7 + 2$

$n = -418$ and $m = -2$

a.

x	1	2	3	4		25	m	51	$m = \underline{\hspace{2cm}}$
y	-2	-5	-8	-11		n	-95	-152	$n = \underline{\hspace{2cm}}$

b.

x	1	2	3	4		m	30	60	$m = \underline{\hspace{2cm}}$
y	3	2	7	12		27	n	292	$n = \underline{\hspace{2cm}}$

c.

x	1	2	3	4		10	15	m	$m = \underline{\hspace{2cm}}$
y	-9	-11	-13	-15		-27	n	-47	$n = \underline{\hspace{2cm}}$

d.

x	1	2	3	4		7	m	46	$m = \underline{\hspace{2cm}}$
y	4	5	6	7		10		n	$n = \underline{\hspace{2cm}}$

e.

x	1	2	3	4		6	10	m	$m = \underline{\hspace{2cm}}$
y	-1	-7	-13	-19		-31	n		$n = \underline{\hspace{2cm}}$

f.

x	1	2	3	4		m	41	70	$m = \underline{\hspace{2cm}}$
y	-12	-14	-16	-18		-70	n		$n = \underline{\hspace{2cm}}$

Problem solving

What is the tenth term? $4x - 5$, $5x - 5$, $6x - 5$

If $y = 5x - 8$ and $x = 2, 3, 4, \dots$, draw a table to show it.



Compare the two examples.

$$-5 + 4$$

$$-5 + 4 = -1$$

$-5 + 4$ is an **algebraic expression**

What is on the
left-hand side of
the equal sign?

$-5 + 4 = -1$ is an **algebraic equation**

What is on the
right-hand
side?

1. Say if it is an expression or an equation.

a. $-4 + 8$

b. $-9 + 7 = -2$

c. $-5 + 10$

d. $-8 + 4 = -4$

e. $-7 + 5$

f. $-15 + 5 - 10$

2. Describe the following:

Example: $-6 + 2 = -4$

This is an expression, $-6 + 2$, that is equal to the value on the right-hand side, -4 .

$-6 + 2 = -4$ is called an equation. The left-hand side of an equation equals the right-hand side.

a. $-8 + 2 = -6$

b. $-15 + 9 = -6$

c. $-11 + 9 = -2$

d. $-5 + 3 = -2$

e. $-8 + 1 = -7$

f. $-4 + 3 = -1$

3. Make use of the variable “a” and integers to create 10 expressions of your own.

Example: $5 + a$

4. Make use of the variable “a” and integers to create 10 equations of your own.

Example: $5 + a = 13$

5. Say if it is an expression or an equation.

Example: $-8 + a$ (It is an expression.)

$-8 + a = -11$ (It is an equation.)

a. $-9 + a = -2$

b. $-3 + a = -1$

c. $-5 + a = -3$

d. $-18 + a$

e. $-12 + a = -3$

f. $-7 + a$

Problem solving

Create 10 examples of algebraic expressions with a variable and a constant. From these create algebraic equations and solve them.



The rule is $-2(n) + 1 =$

Position in sequence	1	2	3	4	5	n
Term	-1	-3	-5	-7	-9	

Write the rule as an expression.

First term: $-2(1) + 1 = -2 + 1 = -1$

Second term: $-2(2) + 1 = -4 + 1 = -3$

Third term: $-2(3) + 1 = -6 + 1 = -5$

Fourth term: $-2(4) + 1 = -8 + 1 = -7$

Fifth term: $-2(5) + 1 = -10 + 1 = -9$

nth term: $-2(n^{\text{th}}) + 1 =$

Note: These expressions all have the same meaning:

$-2n + 1$

$-2 \times n + 1$

$-2 \cdot n + 1$

1. Describe the following in words.

Example: -4, -8, -12, -16, -20, ...

Adding 4 to the previous term.

a. 9; 6; 3; 0; -3

b. 4; 10; 16; 22; 28

c. 7; 14; 21; 28; 35

d. 12; 24; 36; 48; 60

e. 8; 16; 24; 32

f. 6; 16; 26; 36; 46

2. Describe the following sequence using an expression.

Example: -4, -8, -12, -16, -20, ...

Position in sequence	1	2	3	4	5	n
Term	-4	-8	-12	-16	-20	$-3(n) - 1$

First term is $-3(1) - 1$, therefore the rule is $-3(n) - 1$

a. 6; 8; 10; 12; 14

b. 5; 11; 17; 23; 29

c. 4; 13; 22; 31; 40

d. 8; 16; 24; 32; 40

e. 15; 25; 35; 45; 55

f. 4; 7; 10; 13; 16

3. What does the rule mean?

Example: the rule $-2n - 1$ means for the following number sequence:

Position in sequence	1	2	3	4	5	n
Term	-3	-5	-7	-9	-11	$-2n - 1$

(-3 is the first term, -5 is the second term, -7 is the third term, etc.)

a.

Position in sequence	1	2	3	4	5	n
Term	7	13	16	19	23	

b.

Position in sequence	1	2	3	4	5	n
Term	2	10	18	26	34	

c.

Position in sequence	1	2	3	4	5	n
Term						$7n - 5$

d.

Position in sequence	1	2	3	4	5	n
Term						$2n - 3$

e.

Position in sequence	1	2	3	4	5	n
Term	8	17	26	35	44	

f.

Position in sequence	1	2	3	4	5	n
Term	24	37	50	63	86	

Problem Solving

Write a rule for:

On the first day I spend R15, on the second day I spend R30, on the third day I spend R45. How much money would I spend on the tenth if this pattern continued?

I save R15 in January, R30 in February R45 in March. How much money must I save in September if the pattern continues.

Thabo sells one chocolate on Monday, three chocolates on Tuesday and five on Wednesday. How many chocolates will he sell on Friday if the pattern continues?

A farmer plants two rows of maize on the first day 6 rows on the second day and 11 rows on the third day. How many rows must he plant on the 12th day if the pattern continues.

Bongi spends twenty minutes on the computer on day one, thirty minutes on day two and forty minutes on day three. How much time will she spend on the computer on day nine if the pattern continues?





Sequences and algebraic expressions

-5, -9, -13, -17, -21 ...

Describe the rule of this number sequence in **words**.

Subtracting 2 from the previous term.

What does the rule $-4n + 1$ means for the number sequence -3, -7, -11, -15, -19, ... mean?

Write the rule as an **expression**.

First term: $-4(1) + 1 = -3$

Second term: $-4(2) + 1 = -7$

Third term: $-4(3) + 1 = -11$

Fourth term: $-4(4) + 1 = -15$

Fifth term: $-4(5) + 1 = -19$

n^{th} term: $-4(n) + 1$

1. Describe the following in words.

Example: -2, -6, -10, -14, -18, ...

Subtracting 4 from the previous pattern

a. -3; -12; -21; -30; -39

b. -6; -13; -20; -27; -34

c. -3; -5; -7; -9; -11

d. 4; -4; -14; -24; -34

e. -7; -8; -9; -10; -11

f. -8; -12; -16; -20; -24

g. -14; -17; -20; -23; -26

h. -19; -21; -23; -25; -27

i. 9; -2; -13; -24; -35

j. -1; -6; -11; -16; -21

2. Describe the following sequence using an expression.

Example: $-2, -6, -10, -14, -18, \dots$

First term: $-4(1) + 2$

$-4(n) + 2$

a. $2, 4, 5, 6, 10, \dots$

b. $3, 5, 7, 9, 11, \dots$

c. $-8; -20; -32; -44; -56$

d. $-13; -17; -21; -25; -35$

e. $-16; -22; -28; -34; -40$

f. $9; -2; -13; -24; -35$

g. $4; -4; -12; -20; -28$

h. $-3; -12; -21; -30; -39$

i. $-8; -18; -28; -38; -48$

j. $6; -1; -8; -15; -22$

Problem solving

Write three different rules for each:

3; -3; -9; -15; -21

-14; -22; -30; -38; -46

-23; -30; -37; -44; -51

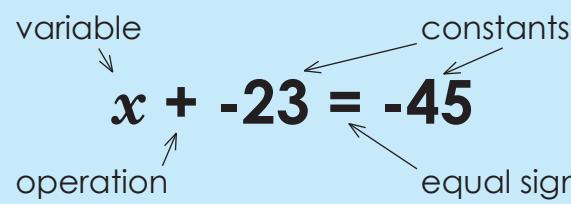
5; 4; 3; 2; 1

19; 7; -5; -17; -29





The algebraic equation



Solving equations

Because an equation represents a balanced scale, it can also be manipulated like one.

Initial equation is $x - 2 = -5$

Add 2 to both sides $x - 2 + 2 = -5 + 2$

Answer $x = -3$

1. Solve for x.

Example: $x - 5 = -9$

$$x - 5 + 5 = 9 + 5$$

$$x = -4$$

a. $x - 12 = -30$

b. $x - 8 = -14$

c. $x - 17 = -38$

d. $x - 20 = -55$

e. $x - 25 = -30$

f. $x - 18 = -26$

g. $x - 6 = -12$

h. $x - 34 = -41$

i. $x - 10 = -20$

j. $x - 25 = -33$

2. Solve for x.

Example: $x + 5 = -2$

$$x + 5 - 5 = -2 - 5$$

$$x = -7$$

a. $x + 7 = -5$

b. $x + 3 = -1$

c. $x + 15 = -12$

d. $x + 17 = -15$

e. $x + 23 = -20$

f. $x + 28 = -13$

g. $x + 10 = -2$

h. $x + 33 = -20$

i. $x + 5 = -10$

3. Solve for x.

Example: $x - 4 + 2 = -7$

$$x - 2 + 2 = -7 + 2$$

$$x = -5$$

a. $x - 3 = -15$

b. $x - 7 = -12$

c. $x - 2 = -5$

d. $x - 5 = -15$

e. $x - 12 = -20$

f. $x - 10 = -25$

g. $x - 23 = -34$

h. $x - 2 = -7$

i. $x - 30 = -40$

Problem solving

Write an equation for the following and solve it.

Five times a certain number minus four equals ninety five.





More on the algebraic equation

$$-2x = 30$$

What does $2x$ mean?

- $2x$ means negative 2 multiplied by x

What is the inverse operation of multiplication?

division

We need to divide $-2x$ by -2 to solve for x .

$$\frac{-2x}{-2} = \frac{30}{-2}$$

$$x = -15$$

Remember you need to balance the scale. What you do on the one side of the equal sign, you must do on the other side as well.



1. Solve for x.

Example: $-3x = 12$

$$\frac{-3x}{-3} = \frac{12}{-3}$$

$$x = -4$$

a. $-5x = 60$

b. $-2x = 24$

c. $-12x = 48$

d. $-7x = 21$

e. $-15x = 60$

f. $-9x = 54$

g. $-5x = 10$

h. $-12x = 36$

i. $-8x = 64$

2. Solve for x.

Example: $-3x - 2 = 10$

$$-3x - 2 + 2 = 10 + 2$$

$$\frac{-3x}{-3} = \frac{-12}{-3}$$

$$x = -4$$

a. $-2x - 5 = 15$

b. $-9x - 4 = 32$

c. $-3x - 3 = 18$

d. $-3x - 2 = 22$

e. $-8x - 4 = 12$

f. $-20x - 5 = 95$

g. $-12x - 5 = 55$

h. $-7x - 3 = 25$

i. $-2x - 2 = 18$

Problem solving

Write an equation and solve it.

- Negative two times y equals negative twelve.
- Negative three times a equals negative ninety nine.
- Negative five times b equals negative sixty.
- Negative four times d equals to forty four.
- Negative three times x equals to thirty.
- Negative two times y equals to sixty four.
- Negative nine times m equal one hundred and eight.
- Negative six times a equals sixty six.
- Negative five times b equals fifteen.
- Negative eight times c equals forty





More algebraic equations

If $y = x^2 + 1$; calculate y when $x = -3$

$$y = (-3)^2 + 1$$

$$y = 9 + 1$$

$$y = 10$$

Test

$$y = x^2 + 1$$

$$10 = (-3)^2 + 1$$

$$10 = 9 + 1$$

$$10 = 10$$

1. Substitute

Example: If $y = x^2 + 2$; calculate y when $x = -4$

$$y = (-4)^2 + 2$$

$$y = 16 + 2$$

$$y = 18$$

Test

$$y = x^2 + 1$$

$$y = (-4)^2 + 2$$

$$y = 16 + 2$$

$$18 = 18$$

a. $y = x^2 + 3$; $x = 3$

b. $y = b^2 + 3$; $b = 4$

c. $y = b^2 + 2$; $x = 4$

d. $y = q^2 + 9$; $q = 5$

e. $y = c^2 + 1$; $c = 7$

f. $y = p^2 + 6$; $p = 2$

g. $y = d^2 + 7$; $d = 9$

h. $y = x^2 + 5$; $x = 3$

i. $y = f^2 + 8$; $f = 10$

j. $y = x^2 + 4$; $x = 12$

2. Solve for x.

Example: If $y = x^2 + \frac{2}{x}$; calculate y when $x = -4$

$$y = -4^2 + \frac{2}{-4}$$

$$y = 16 + \frac{1}{-2}$$

$$y = 15\frac{1}{2}$$

Test

$$y = x^2 + 1$$

$$y = (-4)^2 + 2$$

$$y = 16 + 2$$

$$18 = 18$$

a. $y = x^2 + \frac{2}{x}$; $x = -4$

b. $y = x^2 + \frac{10}{x}$; $x = 15$

c. $y = x^2 + \frac{6}{x}$; $x = -6$

d. $y = x^2 + \frac{5}{x}$; $x = -10$

e. $y = x^2 + \frac{5}{x}$; $x = -10$

f. $y = x^2 + \frac{4}{x}$; $x = -16$

g. $y = x^2 + \frac{3}{x}$; $x = -9$

h. $y = x^2 + \frac{2}{x}$; $x = -8$

i. $y = x^2 + \frac{2}{x}$; $x = -2$

j. $y = x^2 + \frac{1}{x}$; $x = -2$

Problem solving

What is the difference between the value of y in $y = x^2 + 2$, if you first replace y with 3 and then with -3?

y is equal to x squared plus four divided by x if x is equal to eight, solve the equation.

y is equal to p squared plus two divided by p if p is equal to four, solve the equation.

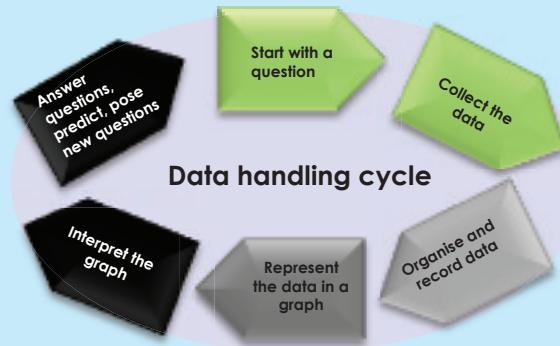
y is equal to b squared plus five divided by b, if b is equal to 10 solve the equation.

y is equal to m squared plus three divided by m, if m is equal to four solve the equation.

y is equal to n squared plus nine divided by n, n is equal to three solve the equation.



Data handling is a cycle. In the worksheet to follow we are going to learn about this cycle. The part we learning about will be in green with some notes.



What will you need to determine the most popular sport in the class?

I will need to ask everyone in the class to select his or her favourite sport.



If we need to know something, we have to start with **posing questions**. What do you think will be the question to ask?

Example:

Before collecting any research data you need to know what question or questions you are asking.

A good way of starting is to come up with a hypothesis. An hypothesis is a specific statement or prediction . The research will find out if it is true or false.

Here are some examples of an hypothesis:

- Everybody in Grade 7 owns a cell phone.
- All Grade 7s understand square roots.
- All Grade 7s like junk food.

1. Where would you look to find data to give you answers to these questions?

a. What is the population of the world?	b. Which learner drinks the most water?
c. What is the rate of population growth in South Africa?	d. What is the population density in this town?
e. What languages are spoken in this area?	f. What is South Africa's most popular food?
g. What is the age structure of the country?	h. What is life expectancy in South Africa?
i. Which country has the youngest population?	j. What are the most popular foods in this school ?

Primary research

when we collect the data ourselves

Secondary research

when we use data collected and analysed by other people

2. Is it always possible to collect data directly from the original source?

continued



Data collection continued

3. In order to collect the data of Question 1, would you do primary or secondary research or both?

4. Let's say you want to know the favourite colours of people at your school, but don't have the time to ask everyone, how will you go about finding the information?

5. How can we make sure that the result is not biased?

If you only ask people who look friendly, you will know what friendly people think!

If you went to the swimming pool and asked people "Can you swim?", you will get a biased answer... probably 100% will say "Yes."

6. How would you design a questionnaire?

A common method of collecting **primary data** is to use a **survey questionnaire**.

Questionnaires come in many forms and are carried out using a variety of methods.

The four main methods of conducting a survey using a questionnaire are:

Face to face

By post

By phone

By internet

There are different ways of designing the questionnaire. You can use:

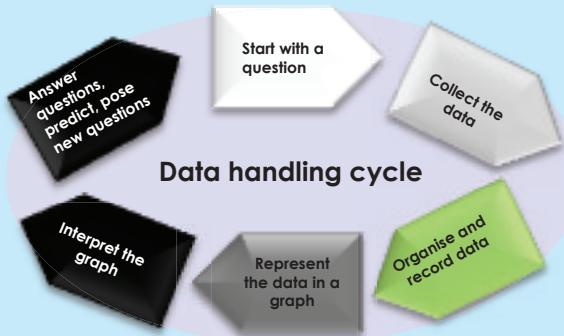
- Yes/No questions
- Tick boxes for multiple choice questions
- Word responses
- Questions that require a sentence to be written.

Problem solving

How much water do learners in the school drink?

- a. Write a hypothesis. .
- b. How will you find the data to prove or disprove the hypothesis? Will this be primary or secondary data?
- c. Find any secondary research data on this topic.
- d. Who should we ask?
- e. What will the data tell us? (What questions will you pose about the data?)
- f. Do you think the data can help us to answer the research question?
- g. Develop some appropriate questions.
- h. Design a simple questionnaire that allows for both both Yes/No type responses and multiple-choice responses.

In the previous worksheet we looked at posing a question and collecting data. The next step in the data handling process is to organise the collected data.



We can organise the data using .

Tallies

$||||\ | = 8$

To **tally** is a way of counting data to make it easy to display in a table. A tally mark is used to keep track of counting.

Frequency tables

A frequency table has rows and columns. When the set of data values are spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we **group** the data into **class intervals** (or groups) to help us organise, analyse and interpret the data.

Stem-and-leaf tables

Stem-and-leaf tables (plots) are special tables where each data value is split into "leaf" (usually the last digit) and a "stem" (the other digits). The "stem" values are listed down, and the "leaf" values go right (or left) from the stem values. The "stem" is used to group the scores and each "leaf" indicates the individual scores within each group.

Example:

Frequency table. The marks awarded for an assignment set for your class of 20 students were as follows:

Colour	Tally	Frequency
Purple		4
Blue		5
Green		3
Red		4

1. These are marks scored by learners writing a test worth 10 marks.

6 7 5 7 7 8 7 6 9 7
4 10 6 8 8 9 5 6 4 8

Present this information in a frequency table.

--	--

Frequency tables for large amounts of data

Example: The best way to summarise the data in a table or graph is to group the possible options together into groups or categories. So, for example, instead of having 100 rows in our table for exam scores out of 100, we may limit it to five rows by grouping the scores together like this: scores between 0-20; 21-40; 41-60; 61-80; 81-100.

Look at this table of exam scores and compile a tally and frequency table with five categories: 0-20, 21-40, 41-60, 61-80, 81-100.

Name	Exam score	Name	Exam score
Denise	55	Elias	65
John	45	Simon	30
Jason	85	Edward	25
Mandla	60	Susan	47
Brenda	79	James	64
Opelo	59	Nhlanhla	77
Lisa	53	Lauren	49
Gugu	90	Tefo	60
Sipho	63	Alicia	46
Lerato	51	Betty	73

Solution

Exam score	Tally	Frequency
0-20		
21-40		2
41-60		10
61-80		6
81-100		2

From this table it is easy to see that most learners scored between 41 % and 60 % for the exam. Two learners failed the exam, because they scored between 0 and 40% and two learners got distinctions, because they scored between 81 and 100%.

2. The number of calls from motorists per day for roadside service was recorded for a month. The results were as follows:

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									



Set up a frequency table for this set of data values, using grouped data, grouped in five groups with intervals of 40.

continued

3. Compile a stem-and-leaf table of the examination data from the example on the previous page.

Example: It will look like this:

Stem	Leaf
2	5
3	0
4	5 6 7 9
5	1 3 5 9
6	0 0 3 4 5
7	3 7 9
8	5
9	0

Now it is easy to see that most learners scored in the 60xs – (most leaves).

Two scored 60 (stem 6 and 2 x leaves of 0), one scored 63, one scored 64 and one scored 65.

Do at home:

1. You collected data by interviewing children in your class regarding their favourite sport.

The results are as follows:

Name	Favourite sport	Name	Favourite sport
Denise	Netball	Elias	Soccer
John	Basketball	Simon	Rugby
Jason	Soccer	Edward	Basketball
Mandla	Cricket	Susan	Soccer
Brenda	Cricket	James	Basket Ball
Opelo	Rugby	Nhlanhla	Rugby
Lisa	Soccer	Lauren	Tennis
Gugu	Tennis	Tefo	Rugby
Sipho	Rugby	Alicia	Soccer
Lerato	Netball	Betty	Netball

Compile a table showing tally and frequency.

2. You recorded the maximum temperatures per day for the past month.

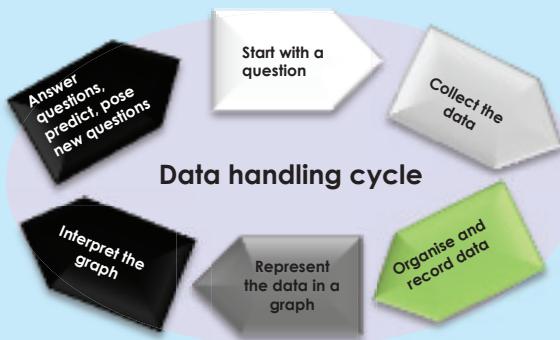
The results are as follows:



28	27	27	26	30	31	30	31	29	28
27	26	24	22	19	19	22	23	24	24
26	27	28	29	30	30	29	28	27	27
27									

- Set up a frequency table for this set of data values, using grouped data, grouped in six groups with intervals of two.
- Compile a stem-and-leaf table of the recorded data.





There are three different types of average generally used to understand data:.

The **range** is the difference between the biggest and the smallest number.

The **mean** is the total of the numbers divided by how many numbers there are.

The **median** is the middle value.

The **mode** is the value that appears the most often.

Example:

Height of learners in cm										
150	152	143	146	135	145	151	139	141	161	
158	148	144	146	155	159	165	149	139	153	
146										

How can we group the data into class intervals (or groups)?



First we need to establish the **range** of the data. The range is the **difference between the biggest and the smallest number**.

Biggest number = 165

Smallest number = 135

Difference = highest number – smallest number

$$\begin{aligned}
 &= 165 - 135 \\
 &= 30
 \end{aligned}$$

So the **range** of this set of numbers is 30.



Height of learners	Tally	Frequency
135-140	///	3
141-145	////	4
146-150	//// /	6
151-155	////	4
156-160	//	2
161-165	//	2

If we want the width of each class interval to be 5, then the number of groups will be: Range \div width of each class = $30 \div 5 = 6$
So we must divide this set of data into six class intervals (or groups).

From the data and the frequency table we can establish that the height of the learners ranges from 135 cm to 165 cm. We also know that 21 learners took part in the survey and that most learners fall into the 146 cm to 150 cm group.

From this data we can also calculate the **mean, median and mode**.

Mean

The **mean** is the total of the numbers divided by how many numbers there are.

This is the most common average that we normally refer to and which we use to calculate our report cards.

135	139	139	141	143	144	145	146	146	146	148	149	150	151	152	153	155	158	159	161	165
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

If we add up all 21 numbers in our data range, we will get 3 125.

$$3\ 125 \div 21 = 148,8$$

Therefore the mean for this data range is 148,8.

Note: the mean average is not always a whole number.



Median

The **median** is the middle value.

In our data range we have 21 records. To work out the median (middle value) we arrange the data from small to big and then count until the middle value.

The median or middle value in our data range will be the 11th number.

135	139	139	141	143	144	145	146	146	146	148	149	150	151	152	153	155	158	159	161	165	
10										11 th number		10									

Therefore the median for this data range is 148.

Mode

The mode is the value that appears the most.

Let us arrange the data from small to big:

135	139	139	141	143	144	145	146	146	146	146	148	149	150	151	152	153	155	158	159	161	165
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

The value that appears the most is 146.

Therefore the mode for this data range is 146.



continued



Summarise data continued

1. Use the data set below and calculate:

3, 13, 7, 5, 21, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29

a. The range

b. The mean

c. The median

d. The mode

2. Sipho wrote seven maths tests and got scores of 68, 71, 71, 84, 53, 62 and 67. What was the median and mode of his scores?

3. What is the mean of these numbers: 18, 12, 10, 10, 25?

4. The mean of three numbers is 8. Two of the numbers are 11 and 7. What is the third number?

5. The temperature in degrees Celsius over four days in July was 21, 21, 19 and 19. What was the mean temperature?

6. What is the mode of these numbers: 75, 78, 75, 71, 78, 25, 75, 29?

7. Five children have heights of 138 cm, 135 cm, 140 cm, 139 cm and 141 cm.
What is the range of their heights?

8. What is the median of these numbers: 2,4; 2,8; 2,3; 2,9; 2,9?

9. The cost of five cakes is R28, R19, R45, R45, R15. What is the median cost?

10. What is the range of this group of numbers: 75, 39, 75, 71, 79, 55, 75, 59?

11. What is the median of these numbers: 10, 3, 6, 10, 4, 8?

Do it on your own.

These are the test results of 20 learners presented in a stem-and-leaf display.

Stem	Leaf
2	5
3	0
4	5 6 7 9
5	1 3 5 9
6	0 0 3 4 5
7	3 7 9
8	5
9	0

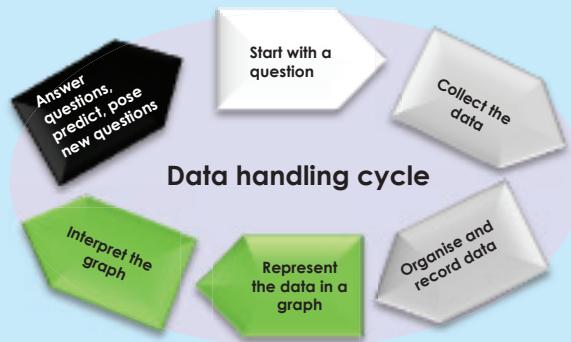
Note:

with an even amount of numbers the median will be the value that would be halfway between the middle pair of numbers arranged from small to big.



1. Use this data to find the :
 - a. Range
 - b. Mean
 - c. Median
 - d. Mode
2. Draw a grouped frequency table showing a tally and frequency column

To record data one can use a bar graph.



Bar graph

A bar graph is a visual display used to compare the frequency of occurrence of different characteristics of data.

This type of display allows us to:

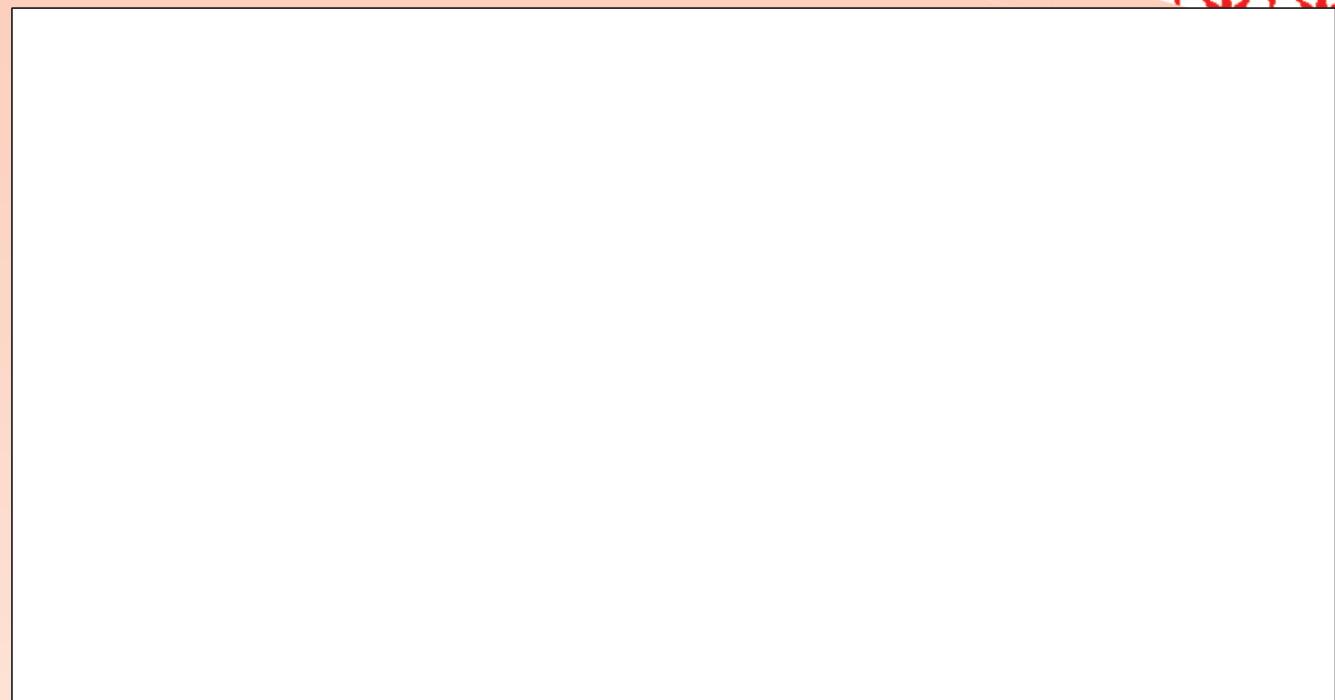
- **compare** groups of data
- make quick **generalisations** about the data.

1. Use the frequency table below to draw a bar graph. Use your bar graph and write three observations regarding the data represented in the graph.

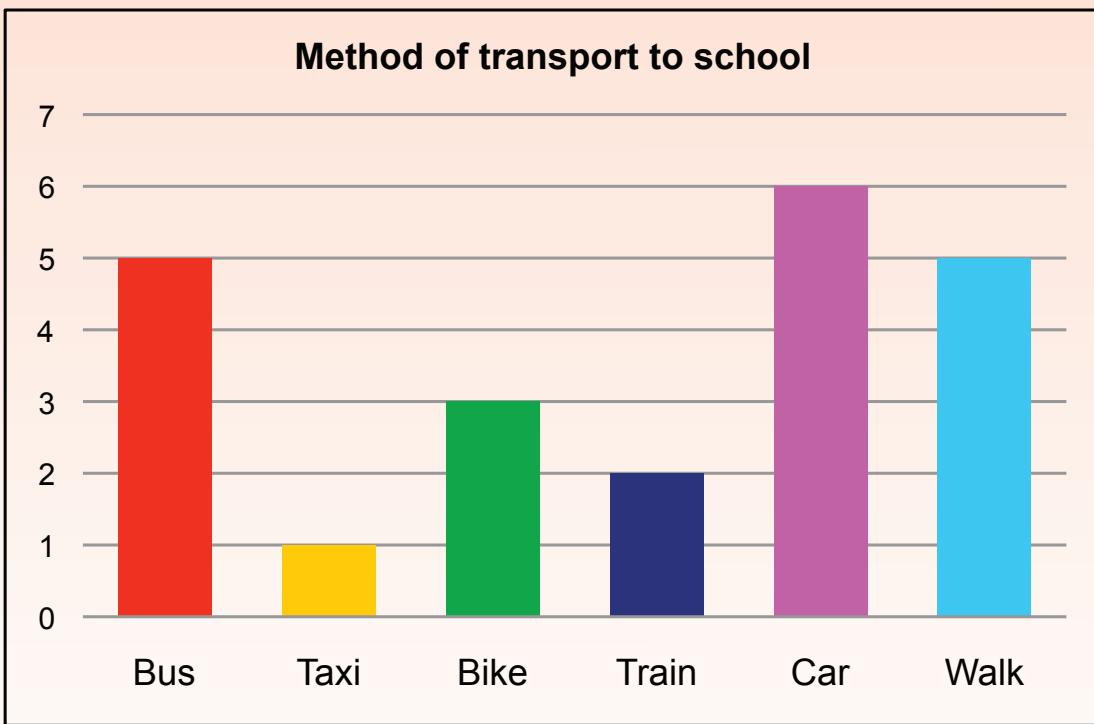
Favourite fruit	Tally	Frequency
Apples	///	3
Oranges	////	4
Grapes	//// /	6
Bananas	////	4
Kiwi	//	2
Strawberries	//	2

Steps to draw a bar graph

1. To draw a bar graph you have to start with your **frequency table**.
2. From the frequency table, decide on **the range and scale** of the frequency data axis (**vertical axis**) and the grouped data axis (**horizontal axis**).
3. Draw the **vertical and horizontal axes** and label them.
4. Write the **graph title** at the top.
5. **Mark the data on the graph** for each data group and **draw the bar**.
6. Add the colour or shading of the bar to **the legend** (key).



2. Critically read and interpret data represented in this bar graph.



Answer the following questions:

a. How many learners are in the class?

continued ➔

b. Which method of transport is the most popular?

c. Which method is the least popular?

d. How many more learners use the bus than the taxi?

e. Why do you think more learners use the bus than the taxi?

f. Do you think most learners live far from or close to the school?

g. What percentage of the learners use public transport?

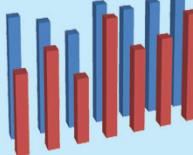
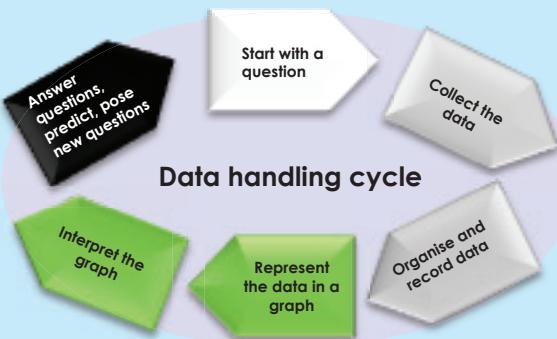
Now try it by yourself

Use the data collected during a survey regarding learners' favourite subjects.

- Compile a frequency table using tallies.
- Draw a bar graph using your frequency table.
- Interpret your graph and write at least five conclusions.

Name	Favourite subject
Peter	Maths
John	Arts
Mandla	History
Bongani	Sciences
Nandi	Sciences
David	Maths
Gugu	History
Susan	Arts
Sipho	Maths
Lebo	Maths
Ann	History
Ben	Maths
Zander	Sciences
Betty	History
Lauren	Arts
Alice	Maths
Veronica	Language
Jacob	Maths
Alicia	History
Thabo	Language

To record data one can use a double bar graph.



Double bar graph

A double bar graph is similar to a regular bar graph, but gives two pieces of related information for each item on the vertical axis, rather than just one.

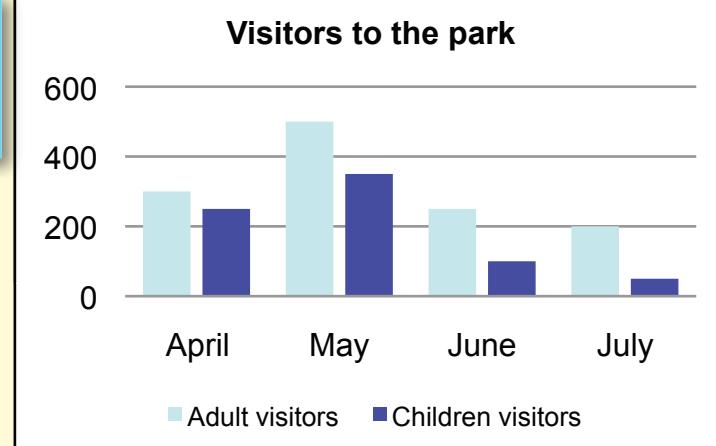
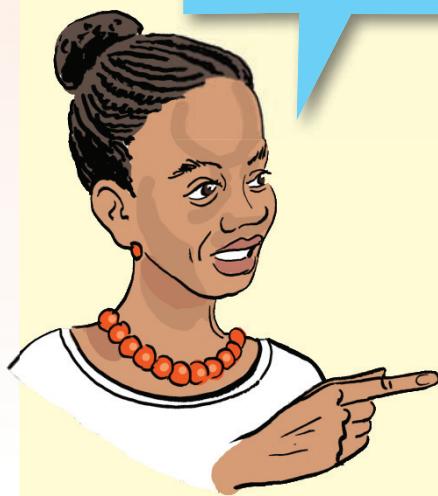
This type of display allows us to compare two related groups of data, and to make generalisations about the data quickly.

Example:

The following frequency table shows the number of adult visitors and child visitors to a park. Construct a **side-by-side** double bar graph for the frequency table.

Visitors to the park				
	April	May	June	July
Adults	300	500	250	200
Children	250	350	100	50

Remember:
the two sets of data on a double bar graph must be related.



1. The results of exam and practical work by a class is shown in the table below.

Name	Practical	Exam	Name	Practical	Exam
Denise	60	65	Elias	55	45
John	63	60	Simon	30	75
Jason	50	50	Edward	65	59
Mathapelo	80	75	Susan	65	75
Beatrix	46	64	Philip	72	75
Opelo	63	53	Ben	46	72
Lisa	51	59	Lauren	31	41
Gugu	67	76	Tefo	75	65
Sipho	81	80	Alicia	63	58
Lorato	78	81	Masa	51	53

a. Compile a frequency table using tallies.

continued

b. Draw a double bar graph comparing the learners' practical marks with their exam marks.

c. Interpret your graph and write down five conclusions.

Do it by yourself

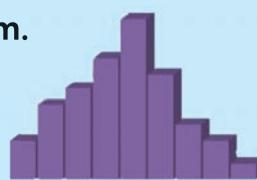
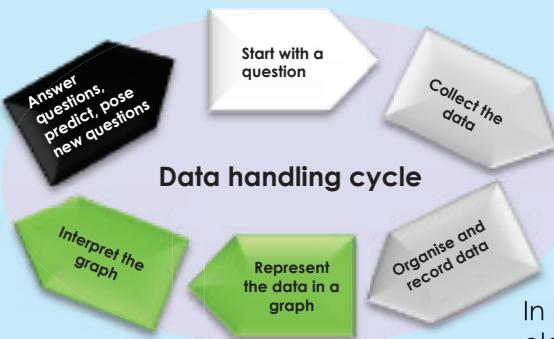
Use the data collected during the survey regarding learners' favourite subjects.

- a. Compile a frequency table using tallies, splitting the different subjects between girls (pink) and boys (blue).
- b. Draw a double bar graph using your frequency table, comparing the preferences between boys and girls.
- c. Interpret your graph and write down at least five conclusions.
- d. How do your conclusions compare with the previous problem solving activity where we used the same data?

Name	Favourite subject	Name	Favourite subject
Peter	Maths	Ann	History
John	Arts	Ben	Maths
Mandla	History	Zander	Sciences
Bongani	Sciences	Betty	History
Nandi	Sciences	Lauren	Arts
David	Maths	Alice	Maths
Gugu	History	Veronica	Language
Susan	Arts	Jacob	Maths
Sipho	Maths	Alicia	History
Lebo	Maths	Thabo	Language



To record data one can use a histogram.



Histogram

A **histogram** is a particular kind of **bar graph** that summarises data points falling in various ranges.

The main difference between a normal bar graph and a histogram is that a bar graph shows you the frequency of each element in a set of data, while a histogram shows you the frequencies of a range of data.

In a histogram the bars must touch, because the data elements we are recording are **numbers** that are **grouped**, and form a **continuous range from left to right**.

Examples of an ordinary bar graph and a histogram

Table A

Favourite colour	Tally	Frequency
Blue	///	3
Red	////	4
Green	//// /	6
Yellow	////	4
Pink	//	2
Purple	//	2

Table B

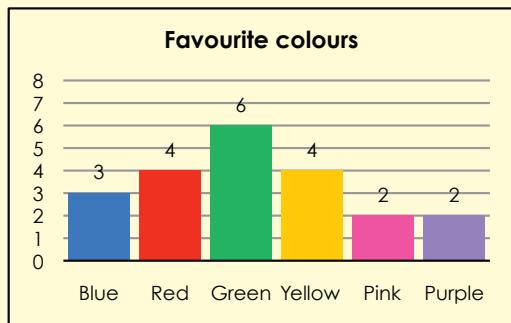
Height of learners	Tally	Frequency
135-140	///	3
141-145	////	4
146-150	//// /	6
151-155	////	4
156-160	//	2
161-165	//	2

What is the difference between the two frequency tables?

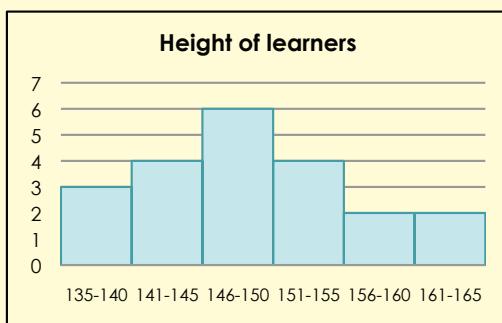
In Table A, the frequency covers individual items (Blue, Red, Green, Yellow, Pink and Purple)

In Table B the frequency covers a range (135 to 165 – divided into smaller groups i.e. 135-140, 141-145, 146-150, 141-155, 156-160 and 161-165)

Bar graph for Table A



Histogram for Table B



In the graph for Table A each bar represents a different attribute. The height of the bar indicates the number of people who indicated that specific colour as their favourite colour.

In the graph for Table B all the bars represent one attribute. The width of the bar represents the range and the height indicates the number of people with the height within that specific range.

Now let us look at how to construct a histogram.

Let us take the following set of numbers: 3, 11, 12, 12, 19, 22, 23, 24, 25, 27, 29, 35, 36, 37, 45, 49
(We can work out that the **mean** is **26.5**, the median is **24.5**, and the **mode** is **12**.)]

In most data sets almost all numbers will be unique and a graph showing how many ones, how many twos, etc. would display data in a meaningful way..

Instead, we group the data into convenient ranges, called **bins**. In this example we are going to group the data in bins with a width of 10 each. Changing the size of the bin will change the appearance of the graph.

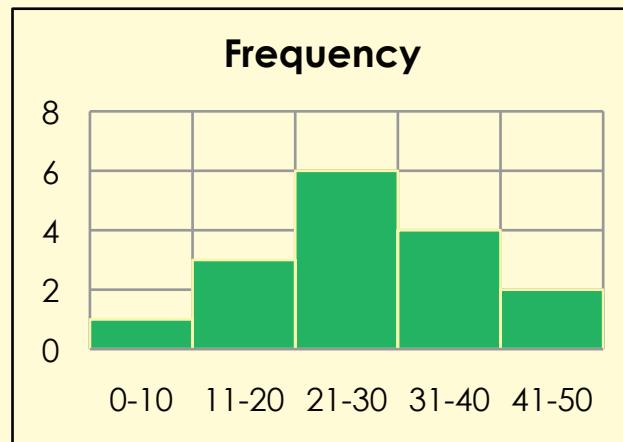
First we draw a frequency table with the data range divided in the different bins.

Data range	Tally	Frequency
0-10		
11-20		
21-30		
31-40		
41-50		

Then we tally the data, placing it in the correct bin.

Data range	Tally	Frequency
0-10	/	1
11-20	///	3
21-30	/ /	6
31-40	///	4
41-50	//	2

Finally we can draw the histogram by placing the bins on the horizontal axes and the frequency on the vertical axes.



Remember we use histograms to summarise large data sets graphically. A histogram helps you to see where most of the measurements are located and how spread out they are.

In our example above we can see that most data falls within the 21-30 bin and that there is very little deviation from the mean of 26,5 and the median of 24,5.

1. Use the following data to draw a histogram.

30, 32, 11, 14, 40, 37, 16, 26, 12, 33, 13, 19, 38, 12, 28, 15, 39, 11, 37, 17, 27, 14, 36

a. What is the mean, median and mode?

c. Complete the frequency table. Make the bins 5 in size ranging from 11 to 40.

d. Draw the histogram.

Problem solving

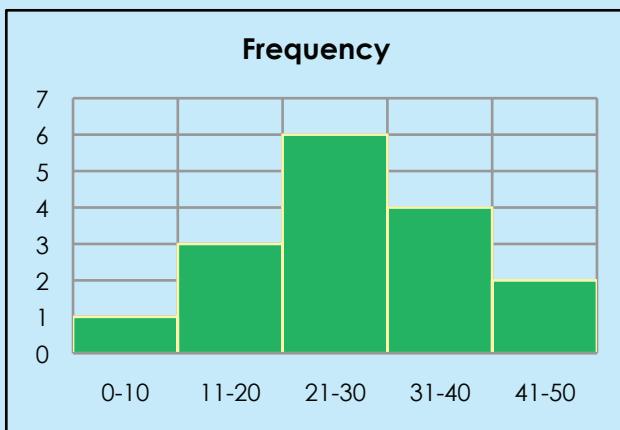
You surveyed the number of times your classmates have travelled to another province. The data you gathered is:

21, 0, 0, 7, 0, 1, 2, 12, 2, 3, 3, 4, 4, 6, 9, 10, 25, 18, 11, 20, 3, 0, 0, 1, 5, 6, 7, 15, 18, 21, 25

Compile a frequency table and then draw a histogram using this data set. Make the bins 3 in size.

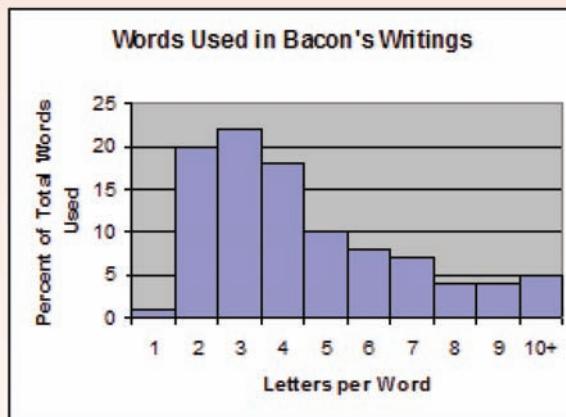
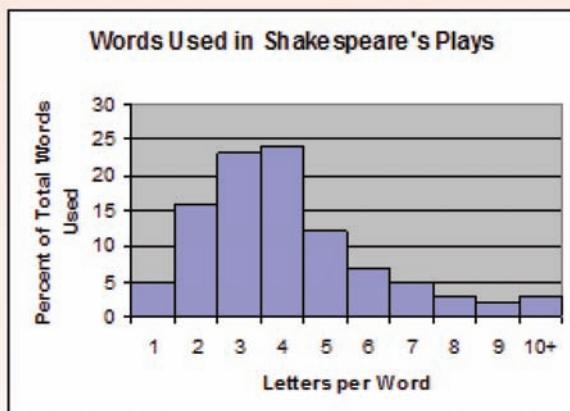
What can you tell us about your survey by looking at the histogram?

Part of the power of histograms is that they allow us to analyse extremely large sets of data by reducing them to a single graph that can show the main peaks in the data, as well as give a visual representation of the significance of the statistics represented by those peaks.



This graph represents data with a well-defined peak that is close to the median and the mean. While there are "outliers," they are of relatively low frequency. Thus it can be said that deviations in this data group from the mean are of low frequency.

1. These two histograms were made in an attempt to determine if William Shakespeare's plays were really written by Sir Francis Bacon. A researcher decided to count the lengths of the words in Shakespeare's and Bacon's writings. If the plays were written by Bacon the lengths of words used should correspond closely.



- a. What percentage of all Shakespeare's words are four letters long?

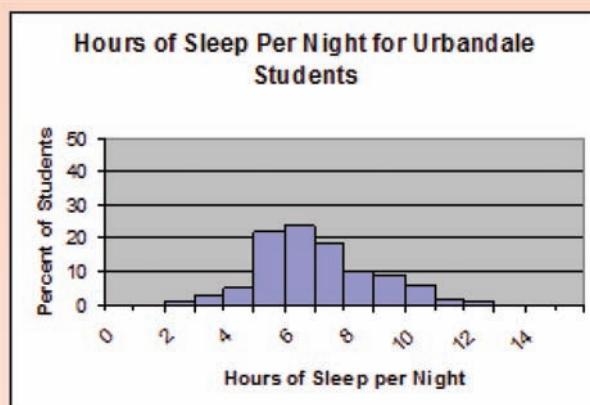
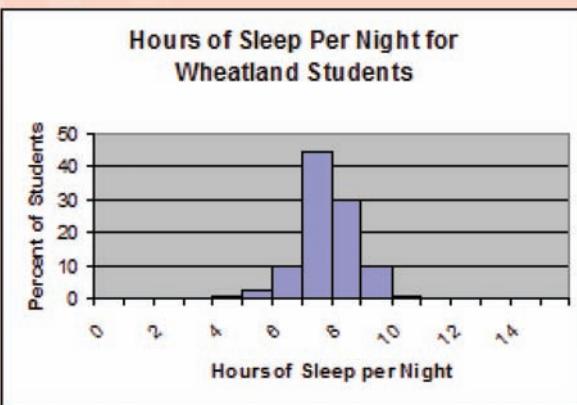
b. What percentage of all Bacon's words are four letters long?

c. What percentage of all Shakespeare's words are more than five letters long?

d. What percentage of all Bacon's words are more than five letters long?

e. Based on these histograms, do you think that William Shakespeare was really just a pseudonym for Sir Francis Bacon? Explain.

2. The two histograms show the sleeping habits of the teens at two different high schools. Maizeland High School is a small rural school with 100 learners and Urbandale High School is a large city school with 3 500 learners.



a. About what percentage of the students at Wheatland get at least eight hours of sleep per night?

b. About what percentage of the students at Urbandale get at least eight hours of sleep per night?

c. Which high school has more students who sleep between nine and ten hours per night?

d. Which high school has a higher median sleep time?

e. Wheatland's percentage of students who sleep between eight and nine hours per night is _____ % more than that of Urbandale.

Problem solving

The table below shows the ages of the actresses and actors who won the Oscar for best actress or actor during the first 30 years of the Academy Awards. Use the data from the table to make two histograms (one for winning actresses' ages and one for winning actors' ages). Use bin widths of ten years (0-9; 10-19; 20-29 etc.)

Year	Age of winning actress	Age of winning actor
1928	22	42
1929	36	40
1930	28	62
1931	62	53
1932	32	35
1933	24	34
1934	29	33
1935	27	52
1936	27	41
1937	28	37
1938	30	38
1939	26	34
1940	29	32
1941	24	40
1942	34	43

Year	Age of winning actress	Age of winning actor
1943	24	49
1944	29	41
1945	37	40
1946	30	49
1947	34	56
1948	34	41
1949	33	38
1950	28	38
1951	38	52
1952	45	51
1953	24	35
1954	26	30
1955	47	38
1956	41	41
1957	27	43

Write a short paragraph discussing what your two histograms reveal.



Notes

Handwriting practice lines for notes.